Handout 5

Math 2130 — Fall 2022

4 December, 2022

1. The matrix A has right eigenvector $\vec{x}$ with eigenvalue λ (that is, $A\vec{x} = \lambda\vec{x}$) and left eigenvector $\vec{y}$ with eigenvalue μ (that is, $\vec{y}^T A = \mu\vec{y}^T$). Assume that $\lambda \neq \mu$. What is $\vec{y}^T\vec{x}$? What is the angle between $\vec{x}$ and $\vec{y}$?

2. What does the previous problem tell you about the angles between eigenvectors of symmetric matrices? Does your answer still apply if there are repeated eigenvalues?

3. If A is an $n \times m$ matrix and $\dim N(A) = k$, what is $\dim N(A^T)$?

4. If A is a square matrix and the λ-eigenspace of A has dimension k, what is the dimension of the λ-eigenspace of A^T?

5. Give an example of a 2×2 matrix that is not diagonalizable (even using complex numbers).

6. Suppose that A is a 2×2 matrix with repeated eigenvalue 3. Let $\vec{x}_1$ be an eigenvector of A with eigenvalue 3 and let $\vec{x}_2$ be any vector in $\mathbb{R}^2$ that is independent of $\vec{x}_1$. Let $X = \begin{pmatrix} \vec{x}_1 & \vec{x}_2 \end{pmatrix}$. Determine as many entries of $X^{-1}AX$ as possible.

7. Find the matrix that rotates space by $\pi/3$ counterclockwise around the vector $\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$. (Hint: find the eigenvectors and eigenvalues first.)

8. Suppose that A and B are 3×3 matrices and $AB = BA$. If A has eigenvalues 1, 2, and 3 and B has eigenvalues 4, 5, and 6, what are the possibilities for the eigenvalues of AB? Explain your answer.

9. Compute the reduced row echelon form of the following matrix:

$$\begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \\ 11 \\ 13 \\ 15 \\ 17 \\ 19 \end{pmatrix} \begin{pmatrix} 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{pmatrix}$$

10. Find a 4×4 Markov matrix A such that the graph of A is connected and $N(A - I)$ is 3 dimensional. (Hint: find the graph of A first.)

11. Suppose that P is a square matrix such that $P^4 = I$. What are the possible eigenvalues of P? Explain both the real and complex eigenvalues.

12. The matrix P performs orthogonal projection onto a subspace of $\mathbb{R}^4$. There is a 4×4 matrix X such that $X^{-1}PX$ is diagonal. Write down all possibilities for $X^{-1}PX$. Remember, $\{\vec{0}\}$ and $\mathbb{R}^4$ are subspaces of $\mathbb{R}^4$.