Name:

Exam 3

Math 2130 — Fall 2022

4 November, 2022

Instructions: Work alone, using only paper and a writing implement. Make sure your name is on every page. Remember that your grade will be based on the understanding you demonstrate, not just the correctness of your answer, so be sure to include comprehensible justification for your work.

1. Find the area of the triangle in the plane with vertices $P = (2, 3)$, $Q = (-1, -1)$, and $R = (4, -2)$.

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2. Compute the determinant of the following matrix, by any method.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 8 & 1 & 2 & 3 \\ 1 & 4 & 2 & 3 & 0 \\ 1 & 2 & 1 & 2 & 1 \end{pmatrix}$$

3. Compute the contribution of the permutation matrix

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

to the ‘big formula’ for the determinant of the matrix A from the last problem.

Instructions: Do at least two of the following six problems. Write your answers on separate paper. Do not forget to write your name on the additional pages you submit.

4. In this problem $A = \begin{pmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{pmatrix}$ is an unknown 3×3 matrix with determinant 4. Let $\vec{x}$ be the following vector

$$\vec{x} = \begin{pmatrix} 5 \\ 0 \\ 7 \end{pmatrix}$$

and let $\vec{b} = A\vec{x}$. Compute the determinant of the matrix $\begin{pmatrix} 2\vec{v}_2 + 3\vec{v}_3 & \vec{b} & \vec{v}_1 \end{pmatrix}$.

5. The following matrix has orthogonal (not orthonormal) columns and *positive* determinant, but some of its entries are unknown. The unknown entries are labelled with an asterisk $(*)$.

$$A = \begin{pmatrix} * & 2 & 4 & * \\ * & * & * & * \\ 1 & 0 & * & -2 \\ * & 3 & * & * \end{pmatrix}$$

The lengths of the columns of A are (in order)

$$3, 10, 6, 5.$$

Write down as many entries of its cofactor matrix as you can.

6. Suppose that A is a square matrix whose entries are all *integers*. For what values of $\det(A)$ does A have an invese whose entries are all integers? Explain your answer.

7. The following matrix has n rows and n columns, following the pattern begun below. Find a pattern relating $\det(A_n)$ to $\det(A_{n-1})$ and $\det(A_{n-2})$. Use it to compute $\det(A_{100})$.

$$A_n = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & \cdots \\ -1 & 2 & -1 & 0 & 0 & \cdots \\ 0 & -1 & 2 & -1 & 0 & \cdots \\ 0 & 0 & -1 & 2 & -1 & \cdots \\ 0 & 0 & 0 & -1 & 2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

8. For what values of c is the following matrix invertible?

$$\begin{pmatrix} -c & 2 & 3 \\ 0 & -6-c & -1 \\ 0 & 6 & -1-c \end{pmatrix}$$

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9. In this problem, A will be the 3×4 matrix shown below and B will be an unknown 4×3 matrix.

$$A = \begin{pmatrix} 2 & 3 & 0 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 1 & 3 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{pmatrix}$$

What we know about B are the determinants of its 3×3 minors:

$$B_{123} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ \cancel{b_{41}} & \cancel{b_{42}} & \cancel{b_{43}} \end{pmatrix} \qquad \det(B_{123}) = -7$$

$$B_{124} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ \cancel{b_{31}} & \cancel{b_{32}} & \cancel{b_{33}} \\ b_{41} & b_{42} & b_{43} \end{pmatrix} \qquad \det(B_{124}) = -1$$

$$B_{134} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ \cancel{b_{21}} & \cancel{b_{22}} & \cancel{b_{23}} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{pmatrix} \qquad \det(B_{134}) = 1$$

$$B_{234} = \begin{pmatrix} \cancel{b_{11}} & \cancel{b_{12}} & \cancel{b_{13}} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{pmatrix} \qquad \det(B_{234}) = 0$$

Calculate the determinant of AB.