

# Exam 1

Math 2130 — Fall 2022

16 September, 2022

Instructions: Work alone, using only paper and a writing implement. Make sure your name is on every page. There are many ways to solve the problems below. Some involve much less calculation than others, so spend some time considering the possible approaches before starting to calculate. Remember that your grade will be based on the understanding you demonstrate, not just the correctness of your answer, so be sure to include comprehensible justification for your work.

1. The following two matrices,  $A$  and  $B$ , include some known and some unknown entries. Unknown entries are marked with an asterisk (\*). Compute all entries of  $AB$  that can be determined from the given information. Place an asterisk (\*) in the entries that cannot be determined. Correct answers do not require additional explanation.

$$A = \begin{pmatrix} 2 & 1 \\ * & 0 \\ -5 & * \end{pmatrix} \quad B = \begin{pmatrix} 0 & -1 & 2 \\ * & 3 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$$

Name: \_\_\_\_\_

2. The matrix on the left has two unknown entries,  $a$  and  $b$ . The matrix on the right is its reduced row echelon form. Determine  $a$  and  $b$ . Show how you arrive at your answer.

$$\text{rref} \begin{pmatrix} 3 & -2 & a \\ 3 & -4 & -4 \\ b & 2 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 8 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \end{pmatrix}$$

$$a = \boxed{\phantom{00}}$$

$$b = \boxed{\phantom{00}}$$

Name: \_\_\_\_\_

3. For what values of  $a$  is the following matrix invertible? Show how you arrive at your answer.

$$\begin{pmatrix} 2 & 1 & 0 \\ -4 & -1 & 1 \\ 2 & 6 & a \end{pmatrix}$$

The matrix is invertible if \_\_\_\_\_.

Name: \_\_\_\_\_

4. Find the matrix  $A$  such that  $AB = C$ , then find  $A^2B$ . Hint: think about row operations. Show how you arrive at your answer.

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 8 & 16 \\ 3 & 9 & 27 & 81 \\ 4 & 16 & 64 & 256 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 7 & 15 \\ 1 & 5 & 19 & 65 \\ 1 & 7 & 37 & 175 \end{pmatrix}$$

$$A = \begin{pmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{pmatrix}$$

$$A^2B = \begin{pmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{pmatrix}$$