Problem 1. Compute $\binom{100}{99}$.

A) 1 B) 99 C) 100 D) 100! E) None of these

Problem 2. For any positive integers n, a, and b with $0 < a < b < n$ we have $\binom{n}{a} < \binom{n}{b}$.

A) True B) False

Problem 3. Suppose that S is a set with n elements and R is an equivalence relation on S. What is the largest the number of equivalence classes of R could possibly be?

A) 1 B) n C) 2^n D) $n!$ E) None of these

Solution. The answer is n. Every equivalence class has to contain at least one element of S. □

Problem 4. How many distinct rearrangements are there of the letters of my name, JONATHAN?

A) 1 B) 8 C) $2 \times 7!$ D) $8!$ E) None of these

Solution. Consider rearrangements where the two n-s are labelled N_1 and N_2 and the two a-s are labelled A_1 and A_2. There are 8! of these, since all of the letters are distinct. Then partition these into equivalence classes based on $\mathrm{N}_1 \equiv \mathrm{N}_2$ and $\mathrm{A}_1 \equiv \mathrm{A}_2$. Each equivalence class has 4 elements so the total number of rearrangements is $8!/4 = 7! \times 8/4 = 7! \times 2$. □

Problem 5. Let S be a set with n elements. How many ways are there to partition S into *two* subsets.

A) 1 B) n C) $2^n - 2$ D) 2^n E) None of these

Solution. One subset is the complement of the other, so this is the same as choosing a non-empty subset of S whose complement is also non-empty. The answer is therefore $2^n - 2$ for $n \geq 2$ and it is 0 when $n = 0$ or $n = 1$. □

Problem 6. Let S be a set. Construct a one-to-one correspondence between the set of partitions of S and the set of equivalence relations on S.

Problem 7. Let S be a set with n elements and let a, b, and c be three positive integers with $a + b + c = n$. You may assume that a, b, and c are all different numbers. Devise a formula using addition, subtraction, multiplication, division, exponentiation, and the factorial for the number of partitions of S into three subsets of sizes a, b, and c.

Solution. $\binom{n}{a}\binom{n-a}{b} = \frac{n!}{a!b!c!}$ □

Problem 8. How does your formula from the last problem change when $a = b \neq c$? What about when $a = b = c$?