Problem 1. Suppose that S is a set, T is a subset of S, and R is an equivalence relation on S. Is $R \cap T \times T$ an equivalence relation on T?
A) Yes B) Sometimes C) No

Problem 2. Suppose that S is a set, T is a subset of S, and R is *not* an equivalence relation on S. Is $R \cap T \times T$ an equivalence relation on T?
A) Yes B) Sometimes C) No

Problem 3. Let $S = \mathbb{R} \times \mathbb{R}$. Let

$$R = \{((a, b), (c, d)) \in S \ : \ ad - bc = 0\}.$$

Is this relation an equivalence relation? Say which properties of an equivalence relation hold and which fail and justify your assertions.

Problem 4. Suppose that A and B are finite sets and that there is at least one bijection from A to B. Which of the following are possible?

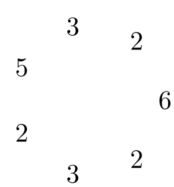
(i) There is a function from A to B that is neither injective nor surjective.
(ii) There is a function from A to B that is injective but not surjective.
(iii) There is a function from A to B that is surjective but not injective.

What if A and B are infinite?

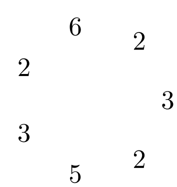
Problem 5. Let S be a set. Which is larger?

A) The number of equivalence relations on S
B) The number of partitions of S
C) They are the same size
D) The question is not well-defined

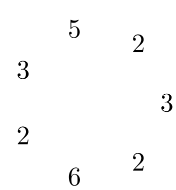
Problem 6. A *cyclic list* is a collection of objects arranged in a circle. A rotation of a cyclic list is still the same list. For example,



and



are representations of the same cyclic list of size 7, but



is a different cyclic list. Compute the number of cyclic lists of size k drawn from a pool of n elements.

Problem 7. Suppose that A and B are finite sets and there are injections $f : A \to B$ and $g : B \to A$. Which of the following can you conclude?
A) $|A| \leq |B|$ B) $|A| \geq |B|$ C) $|A| = |B|$ D) None of these

Problem 8. Let n be an integer. For an integer a, let $[a]$ be the equivalence class of a modulo n. Let S be the set of all equivalence classes of integers modulo n. Prove that the functions

$$\alpha : S \times S \to S$$
$$\mu : S \times S \to S$$

defined by $\alpha([a], [b]) = [a + b]$ and $\mu([a], [b]) = [ab]$ are well defined.

Problem 9. In this problem, the elements of sets are only allowed to be other sets. For any set S, we define the *depth* of S to be the length of the longest chain

$$x_1 \in x_2 \in \cdots \in x_n \in S.$$

(a) List all sets of depths zero, one, and two.

(b) Prove that, for any positive integer n, the number of sets of depth n is equal to 2^m where m is the number of sets of depth $< n$.

(c) How many sets are there of depth 4?