Problem 1. Which of the following proof techniques is most powerful (can be used to prove more things than the others)?

A) Proof by smallest counterexample

B) Proof by induction

C) Proof by strong induction

D) They are all equally powerful

Problem 2. Prove that every integer is either even or odd but not both.

Problem 3. Prove that every rational number can be written a/b where a and b are integers with no common divisor.

Problem 4. Prove that every integer > 1 is divisible by some prime number.

Problem 5. Prove that there are infinitely many prime numbers.

Problem 6. Prove that if a and b are positive integers with no divisors in common then there are integers x and y such that $ax + by = 1$.

Problem 7. Let p be a prime number and let a and b be integers. Prove that if p divides ab then p divides a or p divides b.

Problem 8. Prove that, for any non-negative integer n, either $\sqrt{n}$ is an integer or it is irrational.