Problem 1. Proving $(X \text{ AND } Y) \implies Z$ is equivalent to proving which of the following? Answer according to the following rule:

A) It is equivalent B) It is not equivalent

(i) $(X \implies Z) \text{ AND } (Y \implies Z)$

(ii) $(\text{NOT } Z) \implies ((\text{NOT } X) \text{ OR } (\text{NOT } Y))$

(iii) $(X \implies Z) \text{ OR } (Y \implies Z)$

(iv) $Z \implies (X \text{ AND } Y)$

(v) $(\text{NOT } X) \text{ OR } (\text{NOT } Y) \text{ OR } Z$

Problem 2. Which of the choices below is the logical negation of the sentence below?

$$\forall x, (P(x) \implies S)$$

Note that P is a proposition that depends on x, but S does not depend on x.

(i) $\exists x, ((\text{NOT } P(x)) \text{ AND } S)$

(ii) $\exists x, (P(x) \text{ AND } (\text{NOT } S))$

(iii) $\forall x, (P(x) \implies (\text{NOT } S))$

(iv) $\text{NOT } (\forall y, (P(y) \implies S))$

(v) $(\text{NOT } S) \text{ AND } (\exists x, P(x))$

Problem 3. Fix a non-negative integer n. Formulate a conjecture about the number of permutations of the set $\{1, 2, \dots, n\}$ that have exactly two cycles.

Problem 4. Suppose that a, b, and n are integers such that $0 \le a < n$ and $0 \le b < n$. Prove that $a \equiv b \pmod{n}$ if and only if $a = b$.

Problem 5. Prove that every integer is either even or odd but not both.

Problem 6. Let S be a set, let T be a subset of S, and let $P(x)$ be sentence depending on an element $x \in S$. Suppose that the sentence $\forall x \in S, P(x)$ is false. Determine which of the following are true.

(i) $\exists x \in T, (\text{NOT } P(x))$

(ii) $\forall x \in T, (\text{NOT } P(x))$

(iii) $\text{NOT } \forall x \in T, P(x)$

Problem 7. Same setting as in the last problem. How would you *disprove* the following sentence?

$$(\forall x \in S, P(x)) \text{ AND } (\forall x \in T, P(x))$$

Problem 8. Write a symbolic logical sentence that is equivalent to $\exists! x, P(x)$ without using the ! notation.

Problem 9. Prove that, for any non-negative integer n, either $\sqrt{n}$ is an integer or it is irrational.