Problem 1. Which set is larger?
A) $\mathbf{N}$ B) $\mathbf{R}$ C) They are the same size D) The question doesn't make sense

Problem 2. A finite probability space (or finite sample space) is a pair (S, P) where S is a set and $P : S \to \mathbb{R}$ is a function such that

(i) $P(x) \geq 0$ for all $x \in S$, and
(ii) $\sum_{x \in S} P(x) = 1$.

Which of the following are possible if (S, P) is a finite probability space?

A) $|S| = 0$
B) $|S| = 1$
C) $|S| = \infty$
D) More than one of the above
E) None of the above

Problem 3. Proving $(X \text{ AND } Y) \implies Z$ is equivalent to proving which of the following? Answer according to the following rule:
A) It is equivalent B) It is not equivalent

(i) $(X \implies Z) \text{ AND } (Y \implies Z)$
(ii) $(\text{NOT } Z) \implies ((\text{NOT } X) \text{ OR } (\text{NOT } Y))$
(iii) $(X \implies Z) \text{ OR } (Y \implies Z)$
(iv) $Z \implies (X \text{ AND } Y)$
(v) $(\text{NOT } X) \text{ OR } (\text{NOT } Y) \text{ OR } Z$

Problem 4. Which of the choices below is the logical negation of the sentence below?

$$\forall x, (P(x) \implies S)$$

(i) $\exists x, ((\text{NOT } P(x)) \text{ AND } S)$
(ii) $\exists x, (P(x) \text{ AND } (\text{NOT } S))$
(iii) $\forall x, (P(x) \implies (\text{NOT } S))$
(iv) $\text{NOT } (\forall y, (P(y) \implies S))$
(v) $(\text{NOT } S) \text{ AND } (\exists x, P(x))$

Problem 5. Fix a non-negative integer n. Formulate a conjecture about the number of permutations of the set $\{1, 2, \ldots, n\}$ that have exactly two cycles.

Problem 6. Suppose that a, b, and n are integers such that $0 \leq a < n$ and $0 \leq b < n$. Prove that $a \equiv b \pmod{n}$ if and only if $a = b$.

Problem 7. Prove that every integer is either even or odd but not both.

Problem 8. Let S be a set, let T be a subset of S, and let $P(x)$ be sentence depending on an element $x \in S$. Suppose that the sentence $\forall x \in S, P(x)$ is false. Determine which of the following are true.

(i) $\exists x \in T, (\text{NOT } P(x))$
(ii) $\forall x \in T, (\text{NOT } P(x))$
(iii) $\text{NOT } \forall x \in T, P(x)$

Problem 9. Same setting as last problem. How would you *disprove* the sentence $(\forall x \in S, P(x)) \text{AND} (\forall x \in T, P(x))$?

Problem 10. Write a symbolic logical sentence that is equivalent to $\exists! x, P(x)$ without using the ! notation.

Problem 11. Prove that, for any non-negative integer n, either $\sqrt{n}$ is an integer or it is irrational.