**Problem 1.** How big does a set $S$ have to be to guarantee that for every function $f : S \to \{0, 1\}$, we have $|f^{-1}(0)| > n$ or $|f^{-1}(1)| > n$?

**Problem 2.** Suppose that $m$, $x$, and $y$ are integers and that $a$ and $b$ are natural numbers such that $x^a$ and $y^b$ are both divisible by $m$. How big does $n$ have to be to guarantee that $(x + y)^n$ is divisible by $m$?

**Problem 3.** If $f : A \to B$ is onto then $|A| \geq |B|$.
A) True B) False

**Problem 4.** Let $A$ and $B$ be finite sets. How many functions are there from $A$ to $B$?
A) $|A| + |B|$ B) $|A| \times |B|$ C) $|B|^{|A|}$ D) $|A|^{|B|}$ E) None of these

**Problem 5.** Let $A$ and $B$ be sets. There is an injection from $A$ to $B$ if and only if there is a surjection from $B$ to $A$.
A) True B) False

**Problem 6.** Let $A$ be a set and recall that $2^A$ is the set of all subsets of $A$. Suppose $f : A \to 2^A$ is a function. Which of the following are possible?

A) $f$ is injective
B) $f$ is surjective
C) Both of the above ($f$ is bijective)
D) None of the above

**Problem 7.** Explain the concept of number using only ideas from set theory. What does it mean for one number to be larger than another? What does it mean to be infinite? What is addition? Multiplication? Exponentiation?

**Problem 8.** If a function has the same domain and codomain then it is a bijection.
A) True B) False

**Problem 9.** Let $A$ and $B$ be finite sets. How many *bijections* are there from $A$ to $B$?
A) $|B|^{|A|}$ B) $|A|^{|B|}$ C) $|A|!$ D) $|B|!$ E) None of these

**Problem 10.** Which of the following are true?

A) The composition of two injections is an injection.
B) The composition of two surjections is a surjection.
C) Both of the above
D) None of the above

**Problem 11.** Let $S$ be the set of equivalence classes of integers modulo 7 and let

$$f : S \to S$$

be the function $f([n]) = [n + 3]$. Is this function a permutation?
A) Yes B) No

**Problem 12.** Every permutation of a finite set can be written as a composition of transpositions.
A) Yes B) No