

Problem 1. How many people need to be present to guarantee that two share the same birthday? (Don't forget about leap years.)

- A) 2 B) 365 C) 366 D) 367 E) 732

Problem 2. How many people need to be present to guarantee that three share the same birthday?

- A) 3 B) 368 C) 733 D) 1098

Problem 3. If $f : A \rightarrow B$ is one-to-one then $|A| \leq |B|$.

- A) True B) False

Problem 4. Let A and B be sets. If $|A| \leq |B|$ then every function $f : A \rightarrow B$ is injective.

- A) True B) False

Problem 5. If $f : A \rightarrow B$ is injective but not surjective then $|A| < |B|$.

- A) True B) False

Problem 6. If $f : A \rightarrow B$ is onto then $|A| \geq |B|$.

- A) True B) False

Problem 7. Let A and B be finite sets. How many functions are there from A to B ?

- A) $|A| + |B|$ B) $|A| \times |B|$ C) $|B|^{|A|}$ D) $|A|^{|B|}$ E) None of these

Problem 8. Let n be a positive integer. How many people must be present to guarantee that n of them share the same birthday?

Problem 9. How big does a set S have to be to guarantee that for every function $f : S \rightarrow \{0, 1\}$, we have $|f^{-1}(0)| > n$ or $|f^{-1}(1)| > n$?

Problem 10. Suppose that m , x , and y are integers and that a and b are natural numbers such that x^a and y^b are both divisible by m . How big does n have to be to guarantee that $(x + y)^n$ is divisible by m ?

Problem 11. Let A and B be finite sets. How many *bijections* are there from A to B ?

- A) $|B|^{|A|}$ B) $|A|^{|B|}$ C) $|A|!$ D) $|B|!$ E) None of these

Problem 12. Let A and B be sets. There is an injection from A to B if and only if there is a surjection from B to A .

- A) True B) False

Problem 13. For any set A the inequality $|A| < |2^A|$ holds.

- A) True B) False

Problem 14. Let A be a set and recall that 2^A is the set of all subsets of A . Suppose $f : A \rightarrow 2^A$ is a function. Is it possible for f to be surjective?

- A) Yes B) No

Problem 15. Explain the concept of number using only ideas from set theory. What does it mean for one number to be larger than another? What does it mean to be infinite? What is addition? Multiplication? Exponentiation?