Problem 1. Let S be the set of equivalence classes of congruence modulo 7 on the integers. Draw the graph of the function $f : S \to S$ defined by $f(x) = x + 3$.

Problem 2. Suppose $f : A \to B$ is a function and that there is another function $g : B \to A$ such that $f \circ g = \mathrm{id}_B$. What can you say with certainty about f?
A) f is injective B) f is surjective C) f is bijective D) None of these

Problem 3. What about g?
A) f is injective B) f is surjective C) f is bijective D) None of these

Problem 4. Suppose that $f : A \to B$ and $g : B \to C$ are functions. Assume that both f and g are surjective. Is $g \circ f$ surjective?
A) Yes B) No

Problem 5. Prove that $\binom{n}{k} = \binom{n}{n-k}$ when n and k are non-negative integers such that $0 \leq k \leq n$.

Problem 6. Let S be a finite set. Prove that the number of subsets of S is the same as the number of functions from S to $\{0, 1\}$.

Problem 7. Can a function be one-to-one without being onto?
A) Yes B) No

Problem 8. If $f : A \to B$ is one-to-one then $|B| \geq |A|$.
A) True B) False

Problem 9. If $f : A \to B$ is onto then $|B| \geq |A|$.
A) True B) False

Problem 10. Let A and B be finite sets. How many functions are there from A to B?
A) $|A| + |B|$ B) $|A| \times |B|$ C) $|B|^{|A|}$ D) $|A|^{|B|}$ E) None of these

Problem 11. Let A and B be finite sets. How many *bijections* are there from A to B?
A) $|B|^{|A|}$ B) $|A|^{|B|}$ C) $|A|!$ D) $|B|!$ E) None of these

Problem 12. What does the graph of the identity function on a set S look like?

Problem 13. Prove that $\operatorname{im} f \circ g \subset \operatorname{im} g$. Give an example where $\operatorname{im} f \circ g = \operatorname{im} g$ and an example where $\operatorname{im} f \circ g \neq \operatorname{im} g$.