Problem 1. I saw a parade of elephants and every time I saw a pink elephant, the previous elephant was pink. Which of the following are possible?

A) Every elephant in the parade was pink.

B) At least one elephant in the parade was pink.

C) None of the elephants in the parade were pink.

D) More than one of the above are possible.

E) None of the above is possible.

Problem 2. I saw a parade of elephants and every time I saw 7 pink elephants in a row, the next elephant was pink. What additional information would I have to tell you for you to be *sure* that every elephant in the parade was pink?

A) The first elephant in the parade was pink.

B) There was at least one pink elephant in the parade.

C) There were at least 7 pink elephants in the parade.

D) More that one of the above would be enough information.

E) None of the above is enough information.

Problem 3. Suppose I saw a parade of elephants and every time I saw a pink elephant, *both* the previous and subsequent elephants were pink. Which of the following is the least amount of information necessary to deduce that every elephant in the parade was pink?

A) At least one elephant was pink.

B) The first elephant was pink.

C) The last elephant was pink.

D) Both the first and last elephants were pink.

E) None of the above is enough information.

Problem 4. Suppose that every time I saw 7 consecutive pink elephants, both the elephant preceding and the elephant succeding those elephants were pink. What information would I need to conclude that every elephant was pink?

Problem 5. The division algorithm says that if n is a positive integer then every other integer m can be written as $qn + r$ where q and r are integers and $0 \leq r < n$. Prove the division algorithm by induction.

Problem 6. The *triangular numbers* are the numbers $T_n = \binom{n}{2}$, where $n \geq 0$.

(i) How many triangular numbers are there between 1 and 10?

A) 1 B) 4 C) 7 D) 10

(ii) Find a formula for the sum $T_1 + T_2 + \cdots + T_n$ of the first n triangular numbers and prove it using induction.

Problem 7. Find a formula for

$$\sum_{m=0}^{n} \binom{m}{k}$$

in terms of n and k and prove it by induction on k.