

Problem 1. Suppose that $P(n)$ is a sentence that depends on an integer n . If you can prove that, for all integers n , the sentence $P(n)$ implies $P(n + 1)$ then you can deduce that $P(n)$ is true for all values of n .

A) True B) False

Problem 2. Every nonempty set of positive rational numbers has a least element.

A) True B) False

Problem 3. Let n be a positive integer. For every positive integer m there is exactly one integer r with $0 \leq r < n$ such that $m \equiv r \pmod{n}$.

A) True B) False

Problem 4. The division algorithm says that if n is a positive integer then every other integer m can be written as $qn + r$ where q and r are integers and $0 \leq r < n$. Prove the division algorithm by induction.

Problem 5. Prove the binomial theorem $(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$ using induction.

Problem 6. Find a formula for the first n consecutive cubes

$$1 + 2^3 + 3^3 + 4^3 + \cdots + n^3$$

and prove it by induction.