Problem 1. Let n be a positive integer. Compute

$$\sum_{k=0}^{n}(-1)^k\binom{n}{k} = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n\binom{n}{n}.$$

Problem 2. Let S be a set with n elements. How many ways are there to partition S into *two* subsets.

A) 1 B) n C) $2^n - 2$ D) 2^n E) None of these

Problem 3. Let S be a set with n elements and let a and b be positive integers such that $a + b = n$. How many ways are there to partition S into two subsets of sizes a and b?

A) ab B) $\frac{1}{2}\binom{n}{a}$ C) $\binom{n}{a}$ D) 2^n

Problem 4. Let S be a set with n elements and let a, b, and c be three positive integers with $a + b + c = n$. You may assume that a, b, and c are all different numbers. Devise a formula using addition, subtraction, multiplication, division, exponentiation, and the factorial for the number of partitions of S into three subsets of sizes a, b, and c.

Problem 5. How does your formula from the last problem change when $a = b \neq c$? What about when $a = b = c$?