Problem 1. Say that an integer is *threeven* if it is divisible by 3, *throdd like* 1 if it can be written as $3k + 1$ for some integer k, or *throdd like* 2 if it can be written as $3k + 2$ for some integer k. Prove that an integer is either threeven, throdd like 1, or throdd like 2.

Solution. First observe that 0 is threven since $0 = 3(0)$. This will be the base case for induction.

Now assume for the sake of induction that n is threven, throdd like 1, or throdd like 2. That is there is an integer k such that $n = 3k$, $n = 3k + 1$, or $n = 3k + 2$. Then $n + 1 = 3k + 1$, $n + 1 = 3k + 2$, or $n + 1 = 3k + 3 = 3(k + 1)$. In the first case, $n + 1$ is throdd like 1, in the second it is throdd like 2, and in the third it is threeven. Therefore n is either threven, throdd like 1, or throdd like 2.

We conclude by induction that every natural number is threven, throdd like 1, or throdd like 2. If $n < 0$ then $-n > 0$ so there is an integer k such that $-n = 3k$, $-n = 3k + 1$, or $-n = 3k + 2$. In the first case, $n = 3(-k)$ so n is threven; in the second case, $n = -3k - 1 = -3(k + 1) + 2$ so n is throdd like 2; in the third case, $n = -3k - 2 = -3(k + 1) + 1$ so n is throdd like 1. $\square$

Problem 2. Find a problem involving induction that is challenging and try to solve it. Then ask a precise question about it.