Math 2001-002 Spring 2014
Homework 24

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Problem 1. Let $P(n)$ be a sentence that depends on an integer n. Let m be a positive integer. Suppose the following two statements are true:

(I) If, for some integer k, the sentences $P(k), P(k+1), \ldots, P(k+m)$ are all true then $P(k-1)$ and $P(k+m+1)$ are both true.

(II) There is some integer k such that $P(k), P(k+1), \ldots, P(k+m)$ are all true.

Deduce using induction or proof by smallest counterexample that $P(m)$ is true for all m.

Problem 2. Assume that n and k are non-negative integers. Find a formula for

$$\sum_{m=0}^{n} \binom{m}{k}$$

in terms of n and k and prove it by induction. (Hint: I suggest doing induction on n.)