Unless otherwise noted, complete sentences are required on all problems. All answers must be justified, including those that do not require complete sentences.

Problem 1. (2 points) Which of the following is not logically equivalent to the others?

A) $X \implies Y$

B) $\neg X \vee Y$

C) $\neg Y \implies \neg X$

D) $Y \implies X$

Problem 2. (5 points) Prove or disprove: If S is a set with n elements and R is an equivalence relation on S with m equivalence classes then n is divisible by m.

Problem 3. (8 points) Let Q and R be relations. Define:

$$Q \circ R = \{(x, z) \,:\, \exists y, (y, z) \in Q \text{ and } (x, y) \in R\}$$

(i) (2 points) Suppose that $Q = \{(5, 7), (6, 7)\}$ and $R = \{(1, 5), (1, 6), (2, 6), (3, 6)\}$. Compute $Q \circ R$. Complete sentences are not required.

(ii) (1 point) Draw a picture with elements of the equivalence relations represented as arrows to illustrate Q, R, and $Q \circ R$ from the previous part. Complete sentences are not required.

(iii) (5 points) Prove that a relation R is transitive if and only if $R \circ R \subset R$.

Problem 4. (7 points) On this problem, your answers may use integers, the symbol n, and the operations of addition, subtraction, multiplication, and division. In particular, you may not use binomial coefficients, factorials, or ellipses.

(i) (3 points) Find the coefficient of x^3y^{n-3} in $(x + y)^n$. Complete sentences are not required.

(ii) (4 points) Find the coefficient of $x^3y^4z^{n-7}$ in $(x + y + z)^n$.

Problem 5. (5 points) Prove that, for any real numbers a, b, and c,

$ab = ac$ if and only if $a = 0$ or $b = c$.

Problem 6. (4 points) Consider an infinite parade of elephants, where there is one elephant for each integer. Assume that the n-th elephant is pink if and only if the $(n + 2)$-th elephant is pink. Describe the minimal amount of additional information you would need to conclude that every elephant in the parade is pink.

Problem 7. (5 points) Call a collection of sets $A_1, \dots, A_n$ *triple-wise disjoint* if

$$A_i \cap A_j \cap A_k = \varnothing$$

whenever i, j, and k are pairwise distinct indices between 1 and n. Prove that if $A_1, \dots, A_n$ are triple-wise disjoint then the following formula holds:

$$|A_1 \cup \cdots \cup A_n| = \sum_{i=1}^{n} |A_i| \quad - \sum_{1 \le i < j \le n} |A_i \cap A_j|$$

You may *not* use the inclusion-exclusion formula on this problem.