Problem 1. (8 points) Prove that a number is divisible by both 3 and 7 if and only if it is divisible by 21.

Problem 2. Consider the following sets:

$$X = \{n \in \mathbb{Z} \,:\, 4|n \text{ and } 6|n\}$$
$$Y = \{n \in \mathbb{Z} \,:\, 12|n\}$$

(i) Prove that $Y \subset X$.

(ii) Prove that $X \subset Y$.

Problem 3. Two lists with the same elements and the same length are the same.
True False

Problem 4. Prove the following statement:

For any sets X, Y, and Z we have $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$.

Problem 5. Prove that, for any integers a, b, x, y, and d, if d divides both a and b then d divides $ax + by$.

Problem 6. (i) Suppose that X, Y, and Z are sets. Prove that $X \cup (Y \cup Z) = (X \cup Y) \cup Z$.

(ii) Assume that X, Y, and Z are *pairwise disjoint*. Prove that

$$|X \cup (Y \cup Z)| = |X| + (|Y| + |Z|)$$
$$|(X \cup Y) \cup Z| = (|X| + |Y|) + |Z|.$$

(iii) Prove the associative property of addition for non-negative integers:

$$a + (b + c) = (a + b) + c.$$

Problem 7. Show that the following is a tautology:

$$\Big((V \to W) \wedge (W \to X) \wedge (X \to Y) \wedge (Y \to Z)\Big) \to (V \to Z).$$

(Hint: Don't use a truth table.)