Quiz 5

Math 2001–002, Fall 2016

September 30

Question 1. All non-negative integers can be generated by the following rules:

M1 0 is a non-negative integer.

M2 If x is a non-negative integer then $2x$ is a non-negative integer.

M3 If x is a non-negative integer then $2x + 1$ is a non-negative integer.

Use these rules to generate the non-negative integer 19. At each step, indicate which rule you are using.

Question 2. All positive integers can be generated by the following rules:

N1 1 is a positive integer.

N2 If x is a positive integer then $x + 1$ is a positive integer.

Set up a proof by induction of the following statement, using the above rules for generating all positive integers:

> **Theorem.** For all positive integers n,
>
> $$1^2 + 2^2 + 3^3 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
>
> You do not need to prove the theorem, but you should give a list of statements, one for each of the rules above, that you would demonstrate as part of your proof.

Question 3. Diagram the following statement in a form suitable for proof by induction:

> **Theorem.** For any integer n and any real number x, if $n \geq 0$ then
>
> $$1 + x + x^2 + x^3 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}.$$

Question 4. All integers can be generated by the following rules:

Z1 0 is an integer.

Z2 1 is an integer.

Z3 If x and y are integers then so is $x - y$.

Set up a proof by induction of the following theorem, using the above rules for generating all integers:

> **Theorem.** Every integer is even or odd.

You do not need to prove the theorem, but you should give a list of statements, one for each of the rules above, that you would demonstrate as part of your proof.