Exploration 7/Quiz 3

Math 2001–002, Fall 2016

October 9, 2016

Question 1. In the following theorem and proof, mark sentences that function as assumptions with the letter 'A' at the beginning of the sentence. Mark sentences that function as assertions with the letter 'B' at the beginning of the sentence. Mark claims (assertions that have yet to be demonstrated) with the letter 'C'. For each assumption you mark, indicate how long the the assumption is in force by drawing an arrow.

Question 2. Diagram each of the claims you have found.

Question 3. Whenever an axiom of the integers is used without explanation, mark the spot where it is used with the number of the axiom.

Question 4. Complete the proof. Don't forget to mark the axioms you use as you use them.

Theorem 5. *Every integer is even or odd.*

Proof. Since every integer is obtained by repeatedly adding or subtracting 1 from 0, we can show that every integer is either even or odd by proving the following statements:

(1) 0 is even, and

(2) whenever you add 1 to an integer that is even or odd, you get an integer that is even or odd, and

(3) whenever you subtract 1 from an integer that is even or odd, you get an integer that is even or odd.

We know that 0 is even, because we can find an integer n such that $0 = 2n$ by taking n to be 0. This proves (1).

Now, we would like to prove (2). Suppose that x is an integer that is even or odd. We would like to show that $x + 1$ is also even or odd. The way we will do this is to show that if x is even then $x + 1$ is odd and if x is odd then $x + 1$ is even.

First suppose that x is even. Then, by the definition of evenness, there is an integer y such that $x = 2y$. Therefore $x + 1 = 2y + 1$. This is precisely the form of an odd integer in the definition of oddness, so $x + 1$ is odd.

Now, suppose that x is odd. Then, by the definition of oddness, there is an integer y such that $x = 2y+1$. Therefore $x + 1 = 2y + 2 = 2(y + 1)$. Therefore we can find an integer n such that $x + 1 = 2n$ by taking n to be $y + 1$. By the definition of evenness, this means that $x + 1$ is even. This completes the proof of Step (2).

Next, we will prove (3).