

## Quiz 2

Math 2001-002

Fall 2016

*Instructions:* Do all of the following questions. Don't miss the questions on the back.

**Question 1.** Diagram the following definition:

An integer  $b$  is a *multiple* of an integer  $a$  if there is an integer  $n$  such that  $b = na$ .

**Question 2.** Put the sentences of the following mathematical discussion in order.

- ( ) Suppose that  $a$ ,  $b$ , and  $c$  are integers such that  $b$  is a multiple of  $a$  and  $c$  is a multiple of  $b$ .
- ( ) If  $a$ ,  $b$ , and  $c$  are integers such that  $b$  is a multiple of  $a$  and  $c$  is a multiple of  $b$  then  $c$  is a multiple of  $a$ .
- ( ) By definition, this means that  $c$  is a multiple of  $a$ .
- ( ) **Theorem.**
- ( ) By substitution, we have  $c = mb = mna$ .
- ( ) Q.E.D.
- ( ) By definition of being a multiple, there is an integer  $m$  such that  $c = mb$ .
- ( ) The product of two integers is an integer, so  $mn$  is an integer.
- ( ) *Proof.*
- ( ) Therefore there is an integer  $q$ , namely  $mn$ , such that  $c = qa$ .
- ( ) By definition of being a multiple, there is an integer  $n$  such that  $b = na$ .

Diagram the following sentences in forms suitable for direct proof:

**Question 3.** If a function has derivative zero then it must be a constant. (Suggestion: You may use  $f'$  to stand for the derivative of a function  $f$ .)

**Question 4.** There is no smallest positive rational number.

**Question 5.** Zero is the only real number  $z$  such that  $z + z = z$ .