Exploration 18

Math 2001–002, Fall 2016

October 28, 2016

Question 1. Make a conjecture about the elements of the following set:

$$\{4x + 7y \ : \ x, y \in \mathbb{Z}\}$$

Prove your conjecture.

Question 2. Make a conjecture about the elements of the following set:

$$\{6x + 10y \ : \ x, y \in \mathbb{Z}\}$$

Prove your conjecture.

Question 3. Make a conjecture about the elements of the following set:

$$\{6x + 10y + 15z \ : \ x, y, z \in \mathbb{Z}\}$$

Prove your conjecture.

Question 4. Suppose that a and b are integers. Make a conjecture about the elements of the following set:

$$\{ax + by \,:\, x, y \in \mathbb{Z}\}$$

Your conjecture should state that two sets are equal.

Question 5. Verify that your conjecture is true in the relevant examples considered above. Create new examples to test your conjecture.

Question 6. State another version of your conjecture that makes no reference to sets. Diagram this statement.

Question 7. Diagram your conjecture in a form suitable for direct proof.