Exploration 13

Math 2001–002, Fall 2016

October 14, 2016

Theorem 1. *Let $A_0 = 2$ and $A_1 = 5$ and define A_n for all integers $n \geq 2$ by the recursive formula,*

$$A_n = 5A_{n-1} - 6A_{n-2}.$$

Then $A_n = 2^n + 3^n$ for all integers $n \geq 0$.

Question 2. Set up a proof by induction of the theorem, using the following rules to generate all integers ≥ 0:

N1 0 is an integer ≥ 0.

N2 If x is an integer ≥ 0 then $x + 1$ is an integer ≥ 0.

Question 3. Will you be able to complete this proof? What problems might you encounter when trying to complete it?

Question 4. Set up a proof by *strong* induction of the theorem, using the same rules to generate all integers ≥ 0.

Question 5. Do you encounter the same problems when trying to complete the proof? Do you encounter new ones?

Question 6. Complete the proof using strong induction.

Theorem 7. *Every integer ≥ 18 can be written as $4a + 7b$ for some integers a and b that are ≥ 0.*

Question 8. Answer the same questions about Theorem 7.