

# Exploration 12

Math 2001–002, Fall 2016

October 7, 2016

**Axiom 1.** A *spherical graph* consists of (finitely many) *vertices* and *edges* drawn on a sphere such that

**SG1** The drawing is nonempty (there is at least one vertex).

**SG2** Every edge connects two vertices (the vertices may be the same).

**SG3** The drawing is connected (every pair of vertices is connected by a chain of edges).

**SG4** No two edges cross, and edge passes through a vertex without being connected to that vertex.

The drawing divides the sphere into regions called *faces*.

**Question 2.** Create several examples of spherical graphs and count the numbers of vertices, edges, and faces. Do you observe any relationship between these numbers?

**Question 3.** What sorts of examples would be best to help detect a pattern?

**Question 4.** What are the basic operations you can do to create spherical graphs? What are the smallest things you can do to create a new spherical graph from an old one? Try to find enough different operations so that you can build *any* spherical graph by repeating them.

**Question 5.** How do the operations you mentioned affect the numbers of vertices, edges, and faces?

**Question 6.** State a theorem relating the numbers of vertices, edges, and faces of a spherical graph that is valid for all spherical graphs.

**Question 7.** Prove your theorem.