

Exploration 9/Quiz 4

Math 2001–002, Fall 2016

September 23, 2016

Definition. For any integer $n \geq 0$, a *general configuration of n lines in the plane* (or just a *configuration of lines* for short) is a finite number of lines in the plane such that

Rule 1 no two of the lines are parallel, and

Rule 2 no three of the lines meet at a single point.

We say that two points are in the same *region* of the configuration if they are on the same side of every line in the configuration.

Question 1. Draw a collection of 4 lines that...

- (i) ...satisfies both **Rule 1** and **Rule 2**.
- (ii) ...satisfies **Rule 1** but not **Rule 2**.
- (iii) ...satisfies **Rule 2** but not **Rule 1**.
- (iv) ...satisfies neither **Rule 1** nor **Rule 2**.

Make sure you label which drawing is which!

Question 2. We want to know how many regions are formed by a general configuration of n lines. What kinds of things can we do to discover an answer to this question?

Question 3. Describe rules that can be used to build all general configurations of lines in the plane. Be careful to state your rules precisely, and make sure that you really get every general configuration without generating anything that is not a general configuration.

Question 4. What is the effect on the number of regions of applying each of your rules?

Question 5. Give a way of calculating the number of regions in a general configuration of n lines in the plane. You don't necessarily have to give a formula, but you should explain (prove) why your calculation is correct.

Question 6. Can you find a formula (involving only addition, subtraction, multiplication, division, and integers—no ellipses (...)) for the number of regions in a general configuration of n lines in the plane? Hint: discussion from the beginning of lecture may help.

Question 7. Fill in the blank in the statement of the theorem and complete its proof.

Theorem B. *A general configuration of n lines in the plane has _____ regions.*

Question 8. (Optional) What happens if **Rule 1** or **Rule 2** is suppressed? How will the formula for the number of regions change? You can be as detailed and rigorous as you would like on this question.