Example

Let's agree to call an integer x even if there is an integer z such that $x = 2z$. And let's also agree to call an integer x odd if there is an integer z such that $x = 2z + 1$.

I will now explain why the sum of two even integers is an even integer.

Suppose you had any two even integers, x and y. Since x is even, there must be an integer z such that $x = 2z$. Also, because y is even, there must be an integer w such that $y = 2w$. Therefore

$$x + y = 2z + 2w = 2(z + w).$$

Since $z + w$ is an integer, $2(z + w)$ is an even integer. Thus $x + y$ is an even integer.

You could have picked any two even integers whatsoever for x and y and my reasoning would have shown that $x + y$ is an even integer. Therefore, the sum of any two even integers is even.

Questions

Question 1. Which sentences in the discussion above are assumptions? Which are assertions?

Question 2. Are there any words that offer clues as to whether a sentence is functioning as an assumption or an assertion?

Question 3. What is the role of the sentences "I will now explain why the sum of two even integers is an even integer." and "Now I will explain why the sum of two odd integers is an even integer."? Are they assertions? Are they assumptions?

Question 4. How much of the discussion does each assumption apply to?

Question 5. Are there any *unstated* assumptions in this discussion? Make a list of as many as you can find.

Example

Definition 6. An integer x is said to be *even* if there is an integer z such that $x = 2z$.

Theorem 7. The sum of any two even integers is even.

Proof. Suppose that x and y are even integers. Then, by definition of evenness, there are integers z and w such that $x = 2z$ and $y = 2w$. Therefore

$$x + y = 2z + 2w = 2(z + w).$$

Now, $z + w$ is the sum of two integers, so it is an integer, and therefore $x + y$ is the product of 2 and an integer. Therefore $x + y$ is an even integer, by definition of evenness. Q.E.D.

Questions

Question 8. Write a careful definition of what it means for an integer to be odd.

Question 9. State a theorem about the result of adding two odd integers. State a theorem about the result of adding one odd integer and one even integer.

Question 10. Write proofs of your theorems.

Question 11. In your proof, put each *assumption* in italics. Put each **assertion** in bold.