Problem 1. How many anagrams can be formed from SASSAFRAS?

Solution. Let A be the set of anagrams, R the set of rearrangements of the letters. There is a function $f : R \to A$ that sends a rearrangement to the word that results from that rearrangement. For any $w \in A$, the set $f^{-1}\{w\}$ is precisely the set or arrangements of w. For any $w \in A$, we have

$$|f^{-1}\{w\}| = 4! \times 3!.$$

Therefore we have

$$|R| = \sum_{w \in A} |f^{-1}\{w\}| = 4! \times 3! \times |A|$$

On the other hand, $|R| = 9!$ so $|A| = \frac{9!}{4! \times 3!}$. □

Theorem 2. *Suppose A is a set. The following are equivalent concepts:*

$$\begin{pmatrix} \textit{surjections with} \\ \textit{domain } A \end{pmatrix} \Leftrightarrow \begin{pmatrix} \textit{equivalence} \\ \textit{relations on } A \end{pmatrix} \Leftrightarrow \begin{pmatrix} \textit{partitions} \\ \textit{of } A \end{pmatrix}$$

Solution. If $f : A \to B$ is a set then define

$$R = \{(a, b) \in A \times A \ : f(a) = f(b)\}P = \{f^{-1}\{b\} \ : \ b \in B\} \subset 2^A.$$

Then R is an equivalence relation on A and P is a partition of A.

If R is an equivalence relation on A then let $B = A/R$ and define $f : A \to B$ by

$$f(a) = [a].$$

Also, B is a partition of A.

Finally, if P is a partition of A, let $f : A \to B$ be the function where $f(a)$ is the unique part of P that contains a. Let R be the set of pairs $(a, b) \in A \times A$ such that a and b lie in the same part of P. □

Problem 3. How many ways can a set with 14 elements be partitioned into two subsets of size 3 and four subsets of size 2?

Solution. $\frac{14!}{3!^2 \times 2! \times 2!^4 \times 4!}$ □