Definition 1. Let S be a set. A partition of S is a subset $P \subset 2^S$ such that

(i) $\varnothing$ is not an element of P;

(ii) the elements of P are pairwise disjoint; and

(iii) the elements of P have union S.

Problem 2. Which of the following are partitions of the set $\{\varnothing, 1, 2, 3, 4\}$? Answer A) for yes and B) for no.

(i) $\{\{1, 3\}, \{\varnothing, 2, 4\}\}$

(ii) $\{\{\varnothing, 1, 2, 3, 4\}\}$

(iii) $\{\varnothing, \{1, 4\}, \{2, 3\}\}$

Definition 3. Let S be a set. An equivalence relation R on S is a subset of $S \times S$ with the following properties:

(i) for all $x \in S$, the pair (x, x) is in R;

(ii) for all $x, y \in S$, if $(x, y) \in R$ then $(y, x) \in R$ as well; and

(iii) for all $x, y, z \in S$, if $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$.

An equivalence class of R on S is a nonempty subset V of S such that, for all $x, y \in S$ we have $(x, y) \in V$ if and only if $(x, y) \in R$.

Theorem 4. *If $f : S \to T$ is a function then*

$$|S| = \sum_{t \in T} |f^{-1}\{t\}|$$

Theorem 5. *If P is a partition of a set S then*

$$|S| = \sum_{U \in P} |U|.$$

Theorem 6. *If R is an equivalence relation on a set S then*

$$|S| = \sum_{V \in S/R} |V|.$$

Problem 7. How many distinct rearrangements are there of the letters in the word SASSAFRAS?
A) $4!$ B) $\frac{9!}{3! \times 4!}$ C) $7!$ D) $\frac{9!}{7!}$ E) $9!$

Solution. B) □

Problem 8. How many ways can a set with 4 elements be partitioned into two sets with 2 elements each?
A) $\frac{4!}{2! \times 2! \times 2!} = 3$ B) $\frac{4!}{2! \times 2!} = 6$ C) $\frac{4!}{2!} = 12$ D) $4! = 24$

Solution. A) □

Problem 9. How many ways can a set with 8 elements be partitioned into 4 sets of 2 elements each?
A) $\frac{8!}{2! \times 2! \times 2! \times 2! \times 4!}$ B) $\frac{8!}{4!}$ C) $\frac{8!}{2! \times 2! \times 2! \times 2!}$ D) $\frac{8!}{2!}$ E) $8!$

Solution. A) □