Problem 1. Prove that the sets $\{1, 2, 3\}$ and $\{4, 5, 6\}$ have the same size.

Problem 2. What is a number?

Definition 3. Let f be a relation from a set A to a set B. We say that f is a **bijection** or a **one-to-one correspondence** if the following two properties hold:

(i) for every $a \in A$ there is a unique $b \in B$ such that $(a, b) \in f$, and

(ii) for every $b \in B$ there is a unique $a \in A$ such that $(a, b) \in f$.

Two sets A and B are said to have the same **cardinality** or the same **size** if there is a bijection from A to B.

Theorem 4. *Having the same cardinality is an equivalence relation on sets.*

Definition 5. A **number** is an equivalence class of sets under the equivalence relation of having the same cardinality.

Problem 6. Suppose S is a set with n elements. Prove that the number of subsets of S of size k is equal to the number of subsets of S of size $n - k$.

Problem 7. Suppose that S is a set with n elements. How many bijections are there from S to itself?

Solution. $n!$ $\square$