**Proof Technique 1** (Direct proof)**.** To prove a statement of the form

$$\forall\, x \in S,\ P(x) \implies Q(x)$$

you may use the following template:

(i) Assume $x$ is an element of $S$ and $P(x)$ is true.

(ii) Do logical reasoning under these assumptions to conclude $Q(x)$ is true.

**Problem 1.1.** What should you do if you want to prove $\forall\, x,\ Q(x)$?

**Problem 1.2.** Prove that if $A$, $B$, and $C$ are sets such that $A \subset B$ and $B \subset C$ then $A \subset C$.

**Proof Technique 2** (Example / Counterexample)**.** To prove a statement of the form

$$\exists\, x \in S,\ P(x)$$

construct an element $x$ of $S$ such that $P(x)$ is true.

To *disprove* a statement of the form

$$\forall\, x \in S,\ P(x) \implies Q(x)$$

construct an element $x \in S$ such that $P(x)$ is true and $Q(x)$ is false.

**Problem 2.1.** Prove that it is possible for an integer to be divisible by 2, 3, and 7 and have remainder 3 when divided by 5.

*Solution.* 168 □

**Problem 2.2.** Disprove the assertion that every integer is either prime or composite.

**Proof Technique 3** (Contradiction)**.** To prove a statement $P$:

(i) Assume that $\neg P$ is true.

(ii) Make logical deductions to arrive at a contradiction.

**Problem 3.1.** Prove there are infinitely many prime numbers.