Definition 1. An *equivalence relation* on a set S is a subset $R \subset S \times S$ with the following three properties:

(i) (reflexivity) if $x \in S$ then $(x, x) \in R$;

(ii) (symmetry) if $(x, y) \in R$ then $(y, x) \in R$;

(iii) (transitivity) if $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$.

Problem 2. Let S be the set of all lines in the plane. Let R be the set of all pairs of lines (ℓ_1, ℓ_2) such that ℓ_1 is perpendicular to ℓ_2. Is R an equivalence relation?

A) Yes B) No

Solution. B) □

Problem 3. Let S be the set of all lines in the plane. Let R be the set of all pairs of lines (ℓ_1, ℓ_2) such that ℓ_1 is parallel to ℓ_2. Is R an equivalence relation?

A) Yes B) No

Solution. A) □

Problem 4. Let S be the set of triangles. Let R be the set of pairs (T_1, T_2) where T_1 and T_2 are triangles that have at least *one* angle in common. Is R an equivalence relation on S?

A) Yes B) No

Solution. B) □

Problem 5. Let S be the set of triangles. Let R be the set of pairs (T_1, T_2) where T_1 and T_2 are triangles that share at least *two* angles in common. Is R an equivalence relation on S?

A) Yes B) No

Solution. A) □

Problem 6. Draw all possible equivalence relations on the set $\{1, 2, 3\}$.

Problem 7. What does a picture of an equivalence relation look like?

Definition 8. Let S be a set and R an equivalence relation on S. An *equivalence class* of R is a subset $T \subset S$ such that the following conditions hold:

(i) $R \neq \varnothing$

(ii) $\forall\, x, y \in S, \Big((x \in T \wedge y \in T) \implies (x, y) \in R\Big)$

(iii) $\forall\, x, y \in S, \Big((x \in T \wedge (x, y) \in R) \implies y \in T\Big)$

The *equivalence class of an element* $x \in S$ is $[x] = \{y \in S : (x, y) \in R\}$.

Note that the equivalence class of x depends on the equivalence relation R. If we chose a different equivalence relation, the equivalence class of x would change.