Definition 1. Let A and B be sets. A *relation from A to B* is a set of ordered pairs (a, b) such that $a \in A$ and $b \in B$. In other words, a relation from A to B is a subset of $A \times B$.

If A is a set then a *relation on A* means a relation from A to A.

Problem 2. Which of the following is not a relation from $\{1, 2\}$ to $\{1, 3\}$?

A) $\{(1, 1)\}$

B) $\{(1, 3), (2, 3)\}$

C) $\{(3, 2), (2, 3)\}$

D) $\{(1, 1), (1, 3), (2, 1), (2, 3)\}$

E) More than one of the above

Problem 3. Suppose A and B are sets. Is $\varnothing$ a relation from A to B?
A) Yes B) No C) Depends on A and B

Solution. A) □

Definition 4. Suppose that R is a relation on a set A.

We say that R is . . .	if . . .
reflexive	$\forall\, x \in A,\ (x, x) \in R$
irreflexive	$\forall\, x \in A,\ (x, x) \notin R$
symmetric	$\forall\, x, y \in A,\ (x, y) \in R \implies (y, x) \in R$
antisymmetric	$\forall\, x, y \in A,\ (x, y) \in R \wedge (y, x) \in R \implies x = y$
transitive	$\forall\, x, y, z \in A,\ (x, y) \in R \wedge (y, z) \in R \implies (x, z) \in R$
an *equivalence relation*	if R is reflexive, symmetric, and transitive
a *partial order*	if R is reflexive, antisymmetric, and transitive

Problem 5. Let R be the relation $\{(1, 1), (2, 2), (1, 2)\}$. Is R reflexive?
A) Yes B) No C) On what?

Solution. C) □

Problem 6. Let $A = \{1, 2, 3\}$ and let R be the relation $\{(1, 1), (1, 2), (2, 2)\}$ on A. For each of the following questions, answer A) for Yes and B) for No.

(i) Is R reflexive on A?

(ii) Is R irreflexive on A?

(iii) Is R symmetric on A?

(iv) Is R antisymmetric on A?

(v) Is R transitive on A?