Problem 1. Which of the following are valid ways of proving $X \implies Y$?

A) Suppose X. [Logical reasoning] Conclude Y.

B) Suppose $\neg Y$. [Logical reasoning] Conclude $\neg X$.

C) Suppose X and $\neg Y$. [Logical reasoning] Contradiction.

D) All of the above.

E) Some, but not all, of the above.

Solution. D) □

Problem 2. Which of the following are valid ways of proving $(X \vee Y) \implies Z$?

A) Suppose X. [Logical reasoning] Conclude Z. Suppose Y. [Logical reasoning] Conclude Z.

B) Suppose $\neg Z$. [Logical reasoning] Conclude $\neg X$ and $\neg Y$.

C) Both of the above.

D) None of the above.

Solution. B) □

Problem 3. Prove that 1 is not even.

Proof. Suppose for the sake of contradiction that 1 is even. Then there is an integer c such that $1 = 2c$. But the only solution to the equation $1 = 2c$ is $c = 1/2$, and $1/2$ is not an integer. □

Proof. By definition of divisibility, we must show that there is no integer c such that $2c = 1$. In other words, we must show that, for every integer c, we have $2c \neq 1$.

Observe that every integer c is either ≤ 0 or ≥ 1. We can therefore consider all integers by considering the two possibilities separately:

Suppose that $c \leq 0$. Then

$$2c \leq 2 \times 0 = 0 < 1$$

so $2c \neq 1$.

Suppose that $c \geq 1$. Then

$$2c \geq 2 \times 1 = 2 > 1$$

so $2c \neq 1$.

As every integer c falls into one of the two cases considered above, we deduce that there is no integer c such that $2c = 1$. This means precisely that 1 is not divisible by 2, by the definition of divisibility. □

Problem 4. Which of the two proofs above is better?

Problem 5. Prove that an integer n is even if and only if $n + 1$ is odd.

Problem 6. Prove that an integer n is odd if and only if $-n$ is odd.