

Math 2001 Quiz 14

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Problem 1. Write an English definition meaning “ x is a perfect square”. Then write a symbolic sentence meaning the same thing, using only quantifiers (\forall , \exists), variables, set membership (\in), logical connectives (\wedge , \vee , \neg , \Rightarrow , \Leftrightarrow), the set of integers (\mathbb{Z}), arithmetic (addition, subtraction, multiplication, division, exponentiation), and equality ($=$).

Problem 2. Use summation notation to rewrite the following expression without ellipses:

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2}$$

Problem 3. Compute $\frac{2001!}{1999!}$ and justify your answer.

Problem 4. Use summation notation to rewrite the following expression without ellipses:

$$n^2 = 1 + 3 + 5 + 7 + \cdots + (2n - 1)$$

Problem 5. Prove that the sum of two even integers is even.

Proof. Suppose that a and b are even integers. This means that there are integers x and y such that $a = 2x$ and $b = 2y$. Then $a + b = 2x + 2y = 2(x + y)$, by the distributive law. Since x and y are integers, so is $x + y$, so by the definition of divisibility, $a + b$ is divisible by 2. But to be even means precisely to be divisible by 2, so we conclude that $a + b$ is even.

As a and b were allowed to be arbitrary even integers in the paragraph above, this shows that the sum of two even integers is even. \square

Problem 6. Prove that the sum of two odd integers is even.

Problem 7. Use sum and product notation to rewrite the following expression without ellipses:

$$1 + (1 \times 3) + (1 \times 3 \times 5) + (1 \times 3 \times 5 \times 7) + \cdots + (1 \times 3 \times 5 \times 7 \times \cdots \times (2n + 1))$$

Problem 8. Write a symbolic sentence meaning “ x is not divisible by any perfect square”, using only quantifiers (\forall , \exists), variables, set membership (\in), logical connectives (\wedge , \vee , \neg , \Rightarrow , \Leftrightarrow), the set of integers (\mathbb{Z}), arithmetic (addition, subtraction, multiplication, division, exponentiation), and equality ($=$).

Problem 9. Suppose that x_1, x_2, \dots is a sequence of real numbers. We say that the sequence *converges* to a real number x if, for every positive real number ϵ , there is positive integer N such that, whenever n is a positive integer greater than N , the distance between x_n and x is less than ϵ .

Rewrite this sentence symbolically, using only quantifiers (\forall, \exists), variables, set membership (\in), logical connectives ($\wedge, \vee, \neg, \Rightarrow, \Leftrightarrow$), the set of real numbers (\mathbb{R}), arithmetic (addition, subtraction, multiplication, division, exponentiation), absolute value, and equality and inequalities ($=, \neq, <, \leq, >, \geq$).

Problem 10. Prove that the sum of two integers is even if and only if their difference is even.