Problem 1. Suppose S is a set. Which of the following is equivalent to the sentence "S has at least one element."

A) $\exists x \in S$ B) $\forall x \in S$ C) $\forall x, x \in S$ D) $\exists x, x \in S$

Solution. D) □

Problem 2. Let S and T be sets. Which of the following is equivalent to $S = T$?

A) $\forall x, \big(x \in S \iff x \in T\big)$

B) $\exists x, \big(x \in S \iff x \in T\big)$

Solution. A) □

Problem 3. For any sentences Y and Z, the sentences $Y \wedge Z$ and $\forall x \in \{Y, Z\}, x$ are equivalent.

A) True B) False

Solution. A) □

Problem 4. Which of the following sentences are true?

A) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, y > x$

B) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, y > x$

Solution. A) □

Problem 5. Let $P(x)$ be the sentence "x lives in Los Angeles" and let $Q(x)$ be the sentence "x lives in California". Which of the following sentences are true?

A) $\exists x,\ (Q(x) \implies P(x))$

B) $(\exists x,\ Q(x)) \implies (\exists x,\ P(x))$

C) Both of the above

D) None of the above

Solution. C) □

Problem 6. Let $P(x)$ be the sentence "x lives in Los Angeles" and let $Q(x)$ be the sentence "x lives in California". Which of the following sentences are true?

A) $\forall x,\ (Q(x) \implies P(x))$

B) $(\forall x,\ Q(x)) \implies (\forall x,\ P(x))$

C) Both of the above

D) None of the above

Solution. B) □

Problem 7. Suppose S is a set. Devise a sentence, using only quantifiers, variables, and logical connectives, that means "There are at least two elements of S." (In particular, you are not allowed to refer to the cardinality of S in your sentence.)