**Problem 1.** Restate the sentence "If $x$ is a natural number less than 5 that is divisible by 3 then $x$ is equal to 0 or 3." as a combination of boolean if-then sentences without using any variables. (Constructions like "Every natural number less than 5 is equal to 0 or 3." are not allowed because they are not technically if-then sentences.)

Is this statement true or false? Justify your answer using boolean algebra.

*Solution.* If 0 is divisible by 3 then 0 is equal to 0 or 0 is equal to 3, and
if 1 is divisible by 3 then 1 is equal to 0 or 1 is equal to 3, and
if 2 is divisible by 3 then 2 is equal to 0 or 2 is equal to 3, and
if 3 is divisible by 3 then 3 is equal to 0 or 3 is equal to 3, and
if 4 is divisible by 3 then 4 is equal to 0 or 4 is equal to 3.

□

**Problem 2.** Re-express the sentence "Every prime number is odd." says as a combination of boolean if-then statements, without using any variables. You will need to describe infinitely many if-then statements here, so write enough to illustrate the general pattern and use ellipses to indicate that the pattern continues.

**Definition 3.** Let $n$ be a positive integer. A *list* of *length* $n$ is a sequence of objects $L = (x_1, \dots, x_n)$. We say that $x_i$ is the *$i$-th entry* of the list $L$, or that $x_i$ is in the *$i$-th position* of $L$.

Two lists $L$ and $M$ are said to be *equal* if they have the same length and the $i$-th entry of $L$ is equal to the $i$-th entry of $M$ for every $i$. In other words, if $L = (x_1, \dots, x_n)$ and $M = (y_1, \dots, y_n)$ then $L = M$ if and only if $x_i = y_i$ for every $i = 1, 2, \dots, n$.

**Problem 4.** How many 10-element lists are there if each element is drawn from the same collection of 2 possibilities?

A) 1 B) 10 C) $100 = 10^2$ D) $1024 = 2^{10}$ E) Infinitely many

*Solution.* D) □

**Problem 5.** How many 10-element lists are there if each element is drawn from the same collection of 2 possibilities *and there are no repetitions*?

A) 0 B) 1 C) $90 = 10 \times 9$ D) $1024 = 2^{10}$ E) Infinitely many

*Solution.* A) □

**Problem 6.** How many 0-element lists are there?

A) 0 B) 1 C) Infinitely many

*Solution.* B) □

**Problem 7.** How many truth values of $X_1, \dots, X_{10}$ make the following sentence true?

$$X_1 \vee X_2 \vee \cdots \vee X_{10}$$

A) No solutions

B) Exactly one solution

C) 1023 solutions

D) 1024 solutions

E) Infinitely many solutions