Math 2001 Assignment 42

Your name here

Due Wednesday, December 10

Problem 1. Recall that the Fibonacci numbers are defined the recursive formula below:

$$\begin{aligned} F_0 &= 1 \\ F_1 &= 1 \\ F_n &= F_{n-1} + F_{n-2} && (n \geq 2) \end{aligned}$$

The binomial coefficients $\binom{n}{k}$ are defined for integers n and k and satisfy the following formula (which was proved in class):

$$\begin{aligned} \binom{0}{k} &= \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \\ \binom{n}{k} &= \binom{n-1}{k-1} + \binom{n-1}{k} \end{aligned}$$

Prove that, for all natural numbers n,

$$F_n = \sum_{k=0}^{n} \binom{n-k}{k}.$$

Problem 2. For each $n \in \mathbf{N}$, let $S_n = \{1, \ldots, n\}$. Define G_n to be the set of subsets of S_n that do not contain any consecutive numbers.

(i) Compute $|G_n|$ for $n = 0, 1, 2, 3, 4, 5$.

(ii) Conjecture a relationship between the numbers $|G_n|$ and the Fibonacci numbers. State your conjecture precisely.

Problem 3. Prove that if n is a positive natural number then $\binom{2n}{n}$ is even. (This is the same as Scheinerman, §17, #12.)

Problem 4. Scheinerman, §17, #11. Be careful about $n = 8$.

Problem 5. Scheinerman, §17, #23