Math 2001 Assignment 37

Your name here

Due Friday, November 21

Reading 1. Scheinerman, §25 (again)

Problem 2. Scheinerman, §26, #1. For all parts of this problem, you may assume the codomains of f and g are equal to their ranges.

Problem 3. Scheinerman, §26, #5

Problem 4. Scheinerman, §26, #8

Problem 5. Scheinerman, §26, #11

Problem 6. Suppose $f : A \to B$ and $g : B \to C$ are functions. Prove the following:

(i) If $g \circ f$ is an injection then f is an injection.

(ii) If $g \circ f$ is a surjection then g is a surjection.

Disprove the following:

(iii) If $g \circ f$ is an injection then g is an injection.

(iv) If $g \circ f$ is a surjection then f is a surjection.

Problem 7. Let A be a set and define

$$\begin{aligned} S &= \{C \,:\, C \text{ is a subset of } A\} \\ T &= \{f \,:\, f \text{ is a function from } A \text{ to } \{0,1\}\}. \end{aligned}$$

In this problem we will prove that S and T have the same size by constructing a function from S to T and a function from T to S and showing that they are inverses. Define $\varphi : S \to T$ by the formula $\varphi(C) = f_C$ where

$$f_C(a) = \begin{cases} 0 & a \in C \\ 1 & a \notin C \end{cases}.$$

(i) Verify that φ is a function from S to T. (Hint: You will need to check that if $C \subset S$ then f_C is a function from A to $\{0,1\}$.)

Define $\psi : T \to S$ by the formula

$$\psi(f) = \{a \in A \ : \ f(a) = 0\}.$$

(ii) Verify that ψ is a function from T to S.

(iii) Show that $\psi(\varphi(C)) = C$ for every $C \in S$.

(iv) Show that $\varphi(\psi(f)) = f$ for every $f \in T$.