Math 2001 Assignment 35

Your name here

Due Monday, November 17

Problem 1. Scheinerman, §24, #11

Problem 2. Scheinerman, §24, #14

Problem 3. Scheinerman, §24, #18. Please use the definition from class: A function $f : A \to B$ is a bijection if there is a function $g : B \to A$ such that $g(f(a)) = a$ for every $a \in A$ and $f(g(b)) = b$ for every $b \in B$.

Problem 4. Scheinerman, §24, #19ab

Problem 5. Scheinerman, §24, #23

Problem 6. Let S be a set and k a natural number. In class we introducted the following notation:

(i) S^k is the set of all lists of length k chosen from the elements of S.

(ii) $(S)_k$ is the set of all lists of length k *without repetition* chosen from the elements of S.

Do the following:

(a) Give an example of a set S where S and $(S)_2$ are not disjoint.

(b) Prove that $|S^1| = |S|$ by constructing a bijection.

(c) Prove that $|S^2| = |(S)_2| + |(S)_1|$ by constructing bijections. (Hint: Find a partition of S^2 into two disjoint subsets of sizes $|(S)_2|$ and $|(S)_1|$. This proof was sketched at the end of class; please fill in the details.)

(d) Prove that $|S^3| = |(S)_3| + 3|(S)_2| + |(S)_1|$ by constructing bijections. (Hint: Break S^3 into 5 subsets.)

(e) Conjecture a formula for $|S^4|$ in terms of $|(S)_4|$, $|(S)_3|$, $|(S)_2|$, and $|(S)_1|$.