Math 2001 Assignment 29

Your name here

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Problem 1. Scheinerman, §15, #1

Problem 2. Scheinerman, §15, #2

Problem 3. Let n be an integer. Two integer x and y are said to be congruent modulo n if $n|(x-y)$. For each integer x, let $[x]$ be the equivalence class of x in $\mathbb{Z}$ with respect to congruence modulo n.

If A and B are subsets of $\mathbb{Z}$, define

$$AB = \{ab \,:\, a \in A \wedge b \in B\}.$$

Prove that, for any $x, y \in \mathbb{Z}$, the following formula holds:

$$[x][y] = [xy].$$

(Hint: You may want to refer to the proof that $[x] + [y] = [x+y]$ in the partial solutions to Assignment 28.)

Problem 4. Let $\mathbb{Z}/n\mathbb{Z}$ be the set of equivalence classes of congruence modulo n on the integers. We say that $\mathbb{Z}/n\mathbb{Z}$ is a *field* if $n > 1$ and, for every $[x] \in \mathbb{Z}/n\mathbb{Z}$ other than $[x] = [0]$, there is a $[y] \in \mathbb{Z}/n\mathbb{Z}$ such that $[x][y] = [1]$. (For example, in $\mathbb{Z}/19\mathbb{Z}$, we have $[3][13] = [39] = [1]$.)

For which values of n below is $\mathbb{Z}/n\mathbb{Z}$ a field?

(i) $n = 2$

(ii) $n = 3$

(iii) $n = 4$

(iv) $n = 5$

(v) $n = 6$

(vi) $n = 7$

Can you conjecture a pattern concerning when $\mathbb{Z}/n\mathbb{Z}$ is a field?