Math 2001 Assignment 26

Your name here

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Problem 1. Let A be the set of all polygons drawn in the plane. If P and Q are elements of A, declare that P is *congruent* to Q and write $P \equiv Q$ if P can be transformed to Q using only rigid motions. (You don't actually need to know what a rigid motion is to do this problem, but a rigid motion is a composition of translations, rotations, and reflections.) Prove that $\equiv$ is an equivalence relation on A. You may use the following facts: (i) doing one rigid motion followed by another is a rigid motion; (ii) the reverse of a rigid motion is also a rigid motion; (iii) doing nothing at all is a rigid motion.

Problem 2. (i) Prove that $|$ is a partial order on the set of natural numbers.

(ii) Why is $|$ not a partial order on the set of integers?

(iii) If R is a partial order on a set A then an element x of A is called *maximal* with respect to R if, for every $y \in A$, we have $(y, x) \in R$. An element x of A is called *minimal* with respect to R if $(x, y) \in R$ for every $y \in A$. What are the maximal and minimal elements of $\mathbb{N}$ with respect to the partial order $|$?

(Hint: Some facts from Sections 5 and 6 of the textbook might be useful here.)

Problem 3. Suppose that A is a set and R is a relation on A. Prove that R is transitive if and only if $R \circ R \subset R$. (See Quiz 26 for the definition of $R \circ R$.)