

Math 2001 Assignment 23

Your name here

October 17, 2014

Problem 1. Scheinerman, §12, #21. Note that Problem 12.21c should read $(A \cup B) - C = (A - C) \cup (B - C)$.

Problem 2. Suppose that A and B are finite sets. Prove by induction on the cardinality of A that $|A \times B| = |A| \times |B|$.

Problem 3. In this class, we usually treat numbers and the operations of arithmetic as basic concepts that do not require definition. However, mathematicians still need to be able to define what they mean by arithmetic. Using only the language of set theory, how would you define multiplication of natural numbers?

Problem 4. Prove the following formula for the size of a union of 3 finite sets:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

(Hint: There are many ways to do this problem. The easiest way that I have found begins by writing $A \cup B \cup C = A \cup (B \cup C)$ and then uses the formula for the size of a union of two sets that we discussed in class. You may also want to use the distributive properties for unions and intersections.)