Math 2001 Assignment 22

Your name here

October 17, 2014

Theorem 1. *If n is a natural number and $n \neq 1$ then n is divisible by a prime number.*

Problem 2. Prove that there are infinitely many prime numbers. (Hint: If $p_1, \ldots, p_n$ are prime numbers, consider the number $p_1 \cdots p_n + 1$ and use Theorem 1. Suggestion for a proof by contradiction: Suppose for the sake of contradiction that n is a natural number and that there are n prime numbers. Suggestion for a direct proof: Prove that for every natural number n there are more than n prime numbers.)

Problem 3. A variant of Nim involves two piles instead of one. Each player must choose which pile to draw from. If the sizes of the piles are n amd m, who will win under optimal play?

Theorem 4. *If d is a natural number and e is an integer such that $0 < |e| < d$ then e is not divisible by d.*

Problem 5. The division algorithm states that, for any *positive natural number* d and any integer n there are integers q and r such that $0 \leq r < d$ and $n = qd+r$. You may use Theorem 4 without proving it.

(i) Write a precise, symbolic statement of the division algorithm using quantifiers.

(ii) Prove the division algorithm using induction. (Hint: use bidirectional induction.)