

Math 2001 Assignment 21

Your name here

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Problem 1. Prove by induction that the number of lists with k elements, each of which is drawn, without repetition, from a pool of n choices is

$$\prod_{m=0}^{k-1} (n - m) = n \times (n - 1) \times \cdots \times (n - k + 1).$$

Problem 2. Scheinerman, §9, #15. In Part (d) you should prove your answer is correct. (Hint: Use induction.)

Problem 3. In the game of Nim, two players, A and B , alternate removing either one or two stones from a pile, with A playing first. If a player does not have any legal move then he or she wins.

- (i) If A and B play optimally, who will win when n is divisible by 3? What if n is of the form $3k + 1$ for some integer k ? What if n is of the form $3k + 2$ for some integer k ?
- (ii) In each of the situations where A can win, give a precise description of what move she should make.
- (iii) Prove that the strategy you described in the last part will always work. (Hint: This should be a proof by induction on n .)
- (iv) How will your answer to Part (i) change if A and B are allowed to take between 1 and 3 stones from the pile? What if they can take between 1 and m stones (where m is a natural number and $m \geq 1$)?