1. Exercise 1.8 from Gathmann's notes.

2. Exercise 1.16 from Gathmann's notes.

3. Prove the fundamental theorem of algebra (every polynomial of degree ≥ 1 with complex coefficients has at least one complex solution) from the fact that we discussed in class:

   Theorem 1. *If f is a meromorphic function on $\mathbf{C}$ and γ is a loop in $\mathbf{C}$ not passing through any of the zeroes or poles of f then*

   $$\begin{aligned}(\textit{winding \# of } f \circ \gamma \textit{ around } 0) = \#&(\textit{zeroes of } f \textit{ enclosed by } \gamma) \\ &- \#(\textit{poles of } f \textit{ enclosed by } \gamma).\end{aligned}$$

4. If R is a UFD and $f \in R[x]$ then $\operatorname{cont}(f)$ is defined to be the greatest common divisor of the coefficients of f. In class we proved Gauß's lemma on content:

   Theorem 2. *If* $\operatorname{cont}(f) = \operatorname{cont}(g) = 1$ *then* $\operatorname{cont}(fg) = 1$.

   Prove these corollaries:

   Theorem 3. *Let K be the field of quotients of R. Suppose $f, g \in R[x]$ and* $\operatorname{cont}(f) = 1$ *and $f|g$ in $K[x]$. Then $f|g$ in $R[x]$.*

   Theorem 4. *If R is a UFD then $R[x]$ is a UFD.*