# Topics to Review

## I. Methods of Integration

**i)** Integration by parts
- **a)** How to use L.I.A.T.E. when choosing $u$ and $dv$.
- **b)** The tabular method for repeated integration by parts.
- **c)** Integration by parts for definite integrals.
- **d)** "Looping" (see example 5 on page 516)

**ii)** Trig Integrals
- **a)** What do they look like?
- **b)** Know table 8.3.1 and table 8.3.2.
- **c)** Know the formulas for the integrals of $\sin^2 x, \cos^2 x, \tan^2 x, \sec x$, etc.

**iii)** Trig Substituions
- **a)** What do they look like?
- **b)** Know table 8.4.1 (or be very familiar with the "triangle method")
- **c)** You may have to do a $u$-substitution before the integral has the correct form for a trig substitution.

**iv)** Integrating by Partial Fractions
- **a)** Can be applied only to PROPER rational functions (degree of numerator is less than degree of denominator).
- **b)** If the rational function is improper, you must first use polynomial long division. The remainder will be proper.
- **c)** Know how to form the partial decomposition if you have linear factors, quadratic factors, and what to do if any of those factors are raised to a power.
- **d)** The form of the decomposition will have unknowns in the numerator. How do you solve for the unknowns?
- **e)** Once you've decomposed the integrand into simpler factions DON'T FORGET TO INTEGRATE.

**v)** Improper Integrals
- **a)** Know how to spot both types (an infinite limit of integration or an infinite discontinuity in the integrand).
- **b)** Know how to set up the appropriate limit in each case.
- **c)** Know what is meant by the "convergence" or "divergence" of an improper integral.
- **d)** Know how to evaluate an improper integral: first set up the appropriate limit, then perform the integration (which is now proper), lastly take the limit.

**Practice Problems** 8.2: 1-40; 8.3: 1-52; 8.4: 1-26; 8.5: 1-26; 8.6: 3-30.

## II. $1^{\text{st}}$ order differential equations

- **i)** Be able to identify when a differential equation is first order (only first derivatives appear).
- **ii)** Be able to identify when a first order differential equation is linear.
- **iii)** If the equation is linear, you can always use the method of integrating factors, so know this method well (steps are given on page 585 and also see the differential equations flow chart).
- **iv)** If the equation is nonlinear, the only method we have is separation of variables, so know this method as well (steps are on page 587).
- **v)** Know how to solve a first order initial value problem.
- **vi)** Mixing Problems. These always work the same way. Know how to set up the appropriate first order differential equation and solve the corresponding initial value problem.
- **vii)** Other word problems.

**Practice Problems** 9.1: 27-32, 43-46, 48.

## III. $2^{\text{nd}}$ order linear homogenous differential equations

- **i)** General form of a $2^{\text{nd}}$ order linear homogenous differential equation with constant coefficients (and what all those words mean).
- **ii)** Know Theorem 9.4.1.
- **iii)** Be able to find the auxiliary equation.
- **iv)** Know the form of the general solution in each case:
  - **a)** distinct real roots
  - **b)** one real (double) root
  - **c)** complex conjugate roots
- **v)** Solving initial value problems

**Practice Problems** 9.4: 3-22.

## IV. Sequences

- **i)** Definition of a sequence (definition 10.1.1).
- **ii)** Know how to find the general term, given the first few terms.
- **iii)** Various notations for sequences. You should be able to express any sequence with general term $a_n$ as:
  - **a)** an infinite list, for example $a_1, a_2, a_3, \ldots, a_n, \ldots$
  - **b)** a function, for example $f(n) = a_n$
  - **c)** an expression in brace notation, for example $\{a_n\}_{n=1}^{\infty}$
- **iv)** When does $\lim f(x) = L$ imply $\lim f(n) = L$?
- **v)** Properties of limits (as pertains to sequences), Theorem 10.1.3.
- **vi)** Even/Odd Theorem (10.1.4).
- **vii)** Squeeze Theorem (10.1.5).

**Practice Problems** 10.1: 5-22.

## V. Monotonicity

**i)** Definition of:
  **a)** increasing
  **b)** strictly increasing
  **c)** decreasing
  **d)** strictly decreasing
**ii)** Monotonicity tests and **how to apply them**
  **a)** $a_{n+1} - a_n$
  **b)** $a_{n+1}/a_n$
  **c)** derivative test
**iii)** What does it mean for a property to hold **eventually**?

**Practice Problems** 10.2: 1-18.

## VI. Series

**i)** Definition of a series (definition 10.3.1).
**ii)** What's the difference between a series and a sequence? Don't mix them up.
**iii)** What is the $n$th partial sum, $s_n$? How does this differ from the entire series?
**iv)** What does it mean for a series to converge? to diverge? Be precise (definition 10.3.2).
**v)** Geometric Series
  **a)** Know the general form.
  **b)** Be able to identify a geometric series even if it isn't in general form and find the constants $a$ and $r$.
  **c)** When does a geometric series converge?
  **d)** If it does converge, what is the sum?
  **e)** Converting between fraction form and decimal form for rational numbers.
**vi)** Telescoping Series
  **a)** Know how to spot them.
  **b)** Know when to use Partial Fractions.
  **c)** Know how to find the sum of a telescoping series.
**vii)** The Harmonic Series - know how to spot it, and the fact that it diverges.
**viii)** $p$-Series - what do they look like and when do they converge? Theorem 10.4.5.
**ix)** Know the algebraic properties of series, Theorem 10.4.3.

**Practice Problems** 10.2: 3-14 (this doesn't include $p$-series, see next section).

## VII. The Convergence Tests - For each one, know **when you can apply it** and **what it tells you about the series**.

**i)** The Divergence Test (10.4.1)
**ii)** The Integral Test (10.4.4)
**iii)** The Comparison Test (10.5.1)
**iv)** The Limit Comparison Test (10.5.4)
**v)** The Ratio Test (10.5.5)
**vi)** The Root Test (10.5.6)

**Practice Problems** 10.4: 3-24; 10.5: 5-44.

## VIII. Alternating Series / Conditional Convergence (10.6)

**i)** Definition of an alternating series
**ii)** The Alternating Series Test
  **a)** When can it be applied?
  **b)** What can it tell you?
**iii)** Know that every series must do exactly one of the following, and know what these terms mean:
  **a)** Diverge
  **b)** Converge Conditionally
  **c)** Converge Absolutely
**iv)** Given a series, know how to tell if it converges absolutely, converges conditionally, or diverges.
**v)** The Ratio Test for Absolute Convergence (10.6.5)

**Practice Problems** 10.6: 1-30.

## IX. Maclaurin and Taylor Polynomials (10.7)

**i)** What do the terms "local linear approximation" and "local quadratic approximation" refer to?
**ii)** Know the definition of the $n$th Maclaurin polynomial for $f$:

$$p_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

**iii)** Given a function and an integer $n$ you should be able to compute the $n$th Maclaurin polynomial.
**iv)** Similarly, know the definition of the $n$th Taylor polynomial for $f$ at $x_0$.
**v)** Given a function, a point $x_0$, and an integer $n$ you should be able to compute the $n$th Taylor polynomial at $x_0$.
**vi)** For a nice enough function you should be able to write the $n$th Taylor or Maclaurin polynomial in sigma-notation (e.g. $e^x$ or $\ln x$).
**vii)** Know the definition of the $n$th remainder and the Remainder Estimation Theorem (10.7.4), which is also called the Lagrange error bound.
**viii)** In the Lagrange error bound formula you should know what $M$ is and how to find an appropriate $M$ for a nice enough function (e.g. if $f(x) = \sin x$ or $f(x) = \cos x$ you can take $M = 1$).

**Practice Problems** 10.7: 10-26.

## X. Power Series: Maclaurin and Taylor Series (10.8)

- **i)** Definition of the Maclaurin series for a function $f$.
- **ii)** Definition of a power series in $x$. (Any Maclaurin series is a power series in $x$).
- **iii)** Definition of the Taylor series for a function $f$ about the point $x_0$.
- **iv)** Definition of a power series in $x - x_0$. (Any Taylor series is a power series in $x - x_0$).
- **v)** What are the three possibilities for the convergence of a power series in $x$ (Theorem 10.8.2)?
- **vi)** What are the three possibilities for the convergence of a power series in $x - x_0$ (Theorem 10.8.3)?
- **vii)** In each of these cases know how to find the interval of convergence and radius of convergence using the ratio test for absolute convergence. (Compute $\rho$, set $\rho < 1$, and then solve for $x$). Know how to check whether the endpoints of the interval should be included.

**Practice Problems** 10.8: 25-48.

## XI. Differentiating and Integrating Power Series (10.10)

- **i)** When can a power series be differentiated term-by-term?
- **ii)** Know how to carry out this operation.
- **iii)** When can a power series be integrated term-by-term (indefinitely or definitely)?
- **iv)** Know how to carry out this operation.
- **v)** Given a known Taylor Series, be able to substitute algebraically to find a new Taylor Series and find its radius and interval of convergence.

**Practice Problems** 10.10: 1, 5a, 5b, 6a, 6d, 21b, 23a