Math 1120 **Solutions to Review for the Third Exam** **Spring 2011**

1. Calculate the mean, median and mode for each of the following data sets.

   (a) 4, 8, 3, 3, 5, 3, 5, 7, 1, 9, 0, 2

   Solution: First, order the set: 0, 1, 2, 3, 3, 3, 4, 5, 5, 7, 8, 9. Note that there are 12 points in this data set. Their sum is 50. Thus, the mean is 4.167. The median is 3.5 (between the two middle numbers since the data set has an even number of points). The mode is 3.

   (b) 18, 22, 22, 17, 30, 18, 12

   Solution: The ordered set is: 12, 17, 18, 18, 22, 22, 30. The sum is 139, meaning that the mean is 19.857. The median is 18. The set is bimodal at 18 and 22.

2. Selina claimed that in her class all of the scores on a test were either 100 or 50, so the mean must be 75. Explain whether or not this reasoning is valid.

   Solution: It is entirely possible that everyone in the class received a 100 except one person, who received a 50. In that situation, the mean will clearly not be 75.

3. Use the data in the table to calculate the following:

   To make our lives easier a bit, notice that the data set is already ordered and the number of times each point is in the data set is the "number of workers" column.

   (a) Range.

   Solution: $132,000

   (b) Mean.

   Solution: Notice that there are 40 workers in total and the sum of their salaries is $1,651,000. Thus, the mean salary is $41,275.

   (c) Variance.

   Solution: 473,349,375. Remember, this is the average of the squared differences from the mean. The differences from the mean here are going to be in the thousands, so their squares are going to be big numbers. The average of a set of big numbers will be a big number.

   (d) Standard Deviation.

   Solution: 21,756.59

Salary	Number of Workers
$18,000	2
$22,000	4
$26,000	4
$35,000	3
$38,000	12
$44,000	8
$50,000	4
$80,000	2
$150,000	1

4. The youngest person in a company is 24 years old. The ranges of ages is 34 years. How old is the oldest person in the company?

   Solution: 58

5. To receive an A in a class, Willie needs at least a mean of 90 on five exams. Willie's grades on the first four exams were 84, 95, 86, and 94. What minimum score does he need on the fifth exam to receive an A in the class?

   Solution: So far, Willie has a total of 359 points on exams. He needs to average 90 per exam, meaning that he needs to get a total of 450 points to make the 90 average cutoff. Thus, he needs $450 - 359 = 91$ points on the fifth exam.

6. In a MATH1300 class at CU, the grades on the first exam were as follows:

   92 98 54 78 88 34 76 82 99 87 86 55 67 73 22 89 83 76

   The sorted list is: $\{22, 34, 54, 55, 67, 73, 76, 76, 78, 82, 83, 86, 87, 88, 89, 92, 98, 99\}$.

   (a) Find the mean.

   Solution: 74.39

   (b) Find the median.

   Solution: 80

   (c) Find the mode.

   Solution: 76

   (d) Find the inner quartile range.

   Solution: We find Q_1 at 67 and Q_3 at 88. Thus, the inner quartile range is 21.

   (e) Find the variance.

Solution: 418.90

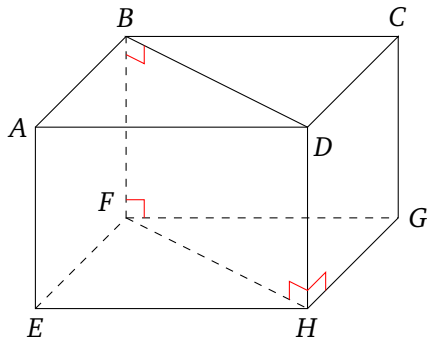
(f) Find the standard deviation.

Solution: 20.47

7. Assume a normal distribution and that the average visit to Professor Slam's office lasts 5 minutes, with a standard deviation of 2 minutes. What percentage of visits lasted longer than 7 minutes? What percentage lasted longer than 9 minutes?

Solution: Remember that, for a normal distribution, 68% of data points will be within one standard deviation of the mean (34% on either side of the mean). See page 647 in your text. You should remember the numbers corresponding to those standard deviations: 34%, 13.5%, and 2.4%. In this case, we know that 68% of students will spend between 3 and 7 minutes in Professor Slam's office. Half of the remaining 32% will spend less than 3 minutes and half will spend more than 7 minutes. Thus, 16% of students will spend more than 7 minutes in office hours. If we do two standard deviations instead of one, we know that a total of 95% of students will spend between 1 and 9 minutes in office hours, again half of the remaining spending more time and half spending less. Thus, 2.5% of students spend more than 9 minutes in Professor Slam's office.

8. The following figure is a box in which the top and bottom are rectangles and $\overline{BF}$ perpendicular to plane FGH. Answer the following.



(a) Find the intersection of $\overline{BH}$ and plane DCG.

Solution: H

(b) Name two pairs of perpendicular planes.

Solution: One pair is FGH and CGH. Another pair is AEH and FEH.

(c) Name two lines that are perpendicular to plane EFH.

Solution: $\overline{AE}$ and $\overline{CG}$. (There are 4 such lines.)

(d) What is the measure of the angle $\angle DHE$?

Solution: 90 degrees

(e) Name an angle in the diagram that is NOT 90 degrees.

Solution: $\angle FHE$.

(f) Name two lines that do not intersect.

Solution: $\overline{FG}$ and $\overline{EH}$.

9. Consider a standard wall clock. Answer the following questions.

This is very close to a problem on the homework. The first two questions will help us answer the others.

(a) How far does the hour hand travel (in degrees) in one minute?

Solution: The hour hand travels 1/12 of the clock in one hour, and thus travels 1/12th of 360° in one hour. Thus, it travels

$$\frac{1}{12} \cdot 360^\circ = 30^\circ$$

in one hour. If we divide this by 60 minutes, we will now know how many degrees the hand travels per minute.

$$\frac{1}{60} \cdot 30^\circ = 0.5^\circ$$

Thus, the hour hand travels half a degree every minute.

(b) How far does the minute hand travel (in degrees) in one minute?

Solution: Since the minute hand travels 360° degrees in one hour, it must travel

$$\frac{1}{60} \cdot 360^\circ = 6^\circ$$

per minute.

(c) Using your previous answers, what is the angle measure between the hour and minute hands at 3:30 PM?

Solution: The easiest way to reason to a solution here is to realize that the hour hand is halfway between the 3 and 4, while the minute hand is pointing directly at the 6. In other words, the hour hand has 2.5 hours until it will be in the location where the minute hand is currently. 2.5 hours is exactly 150 minutes. Since the hour hand will travel

$$150 \cdot 0.5^\circ = 75^\circ$$

in 150 minutes, that is the number of degrees we are looking for.

(d) What is the angle measure between the hour and minute hands at 3:15 PM?

Solution: It has been 15 minutes since the hour hand was where the minute hand is now. In 15 minutes, the hour hand travels 7.5°.

10. For each description, draw an example of a curve/shape matching that description.

    (a) A simple non-closed curve.

    Solution: See the top figure in Table 11-4

    (b) A simple, closed, convex polygon.

    Solution: See the second to last figure in Table 11-4

    (c) A closed and concave curve.

    Solution: There are several examples in Table 11-4. For instance, the last.

    (d) A curve that is neither simple nor closed.

    Solution: Think of a loop made from a length of string, where the string only crosses itself once and the ends don't touch. It is not simple (because it crosses itself) and it is not closed (since it starts and stops at different places).

    (e) A simple, closed, convex curve that is not a polygon.

    Solution: A circle.

11. Notice that a pentagon has only two diagonals that intersect at a given vertex. (Draw this to convince yourself. It is not immediately obvious.) Determine how many diagonals intersect at a given vertex in each of the following polygons.

    (a) Hexagon.

    Solution:

    (b) Decagon.

    Solution:

    (c) 20-gon.

    Solution:

    (d) n-gon for arbitrary positive integers n.

    Solution: