Math 1120 **Review for the Second Exam** **Spring 2011**

1. Suppose all names consist of a first name, a middle initial, a last name and no other name, so the initials of any name consist of three letters. Assume that any letter is just as likely as any other to be an initial, and initials can be repeated.

   (a) What is the same space of all *last name* initials?

   Answer: $\{A, B, C, \ldots, Z\}$

   (b) Write four examples of elements from the sample space of all possible initials (consisting of all three names).

   Answer: $A.F.G.$, $B.M.N.$, $J.K.L.$, $C.D.Z.$

   (c) What is the probability that two people share the same first initial?

   Answer: $\dfrac{1}{26}$

   (d) What is the probability that two people share all three initials in the same order?

   Answer: $\left(\dfrac{1}{26}\right)^3$

   (e) What is the probability that the first person has a middle initial that occurs earlier in the alphabet than H (not including H)?

   Answer: $\dfrac{7}{26}$

   (f) What is the probability that the second person has a middle initial that is a vowel?

   Answer: $\dfrac{5}{26}$

   (g) What is the probability that the first person has a middle initial that occurs earlier in the alphabet than H (not including H) AND the second person has a middle initial that is a vowel?

   Answer: $\left(\dfrac{7}{26}\right)\left(\dfrac{5}{26}\right)$ (Note that the two events are independent.)

   (h) What is the probability that the first person has a middle initial that occurs earlier in the alphabet than H (not including H) OR the second person has a middle initial that is a vowel?

   Answer: $\dfrac{7}{26} + \dfrac{5}{26} - \left(\dfrac{7}{26}\right)\left(\dfrac{5}{26}\right) = \dfrac{277}{676} \approx 0.4098$

2. Suppose that next week, it will rain one day and snow on two days. The other four days will be sunny. (This accounts for the entire week.)

   (a) What are the odds for it snowing on Tuesday?

   Answer: 2:5

   (b) What are the odds against any precipitation (rain or snow) on Tuesday?

   Answer: 4:3

3. A bag contains n red and yellow marbles.

   (a) If a marble is chosen at random, the probability that it is red is $\frac{2}{5}$. If there are 15 yellow marbles in the bag, how many red marbles are there?

   Answer: 10

   (b) If a marble is chosen at random, the probability that it is red is $\frac{2}{5}$. Suppose we double the number of red marbles in the bag, but keep the number of yellow marbles the same. What is the probability a randomly chosen marble is red now?

   Answer: $\dfrac{4}{7}$

   (c) Let r be the number of red marbles. Suppose we add five blue marbles to the bag. What is the probability that a randomly chosen marble is yellow? (Your answer may be in terms of n and r.)

   Answer: $\dfrac{n-r}{n+5}$

4. Suppose that the probability that a child born is a girl is $\frac{1}{2}$.

   (a) Assuming no twins, what is the probability that a family with three children will have all girls?

   Answer: All of the outcomes are equally likely, and the event of having all girls is given by a single outcome. Really, this just means that we multiply the probabilities for the branches in the probability tree leading down to GGG (for three girls). $\left(\dfrac{1}{2}\right)^3 = \dfrac{1}{8}$

   (b) Assuming no twins, what is the probbaility that a family with three children will have exactly two girls?

   Answer: This problem differs from the latter in that there is more than a single way to achieve the desired outcome. (For instance, GBG and GGB.) The probability of any specific outcome is the same as before, but we now have ${}_3C_2$ ways to achieve the desired outcome. $\left(\dfrac{1}{2}\right)^3 {}_3C_2 = \left(\dfrac{1}{2}\right)^3 * 3 = \dfrac{3}{8}$

(c) Assuming no twins, what is the probability that a family with three children will have two or more girls?

Answer: We know that the probability of having exactly two girls is $\frac{3}{8}$. We also know that the probability of having all girls is $\frac{1}{8}$. If we add these probabilities together, we get $\dfrac{1}{2}$.

5. Suppose that the probability that a child born is a girl is $\frac{2}{3}$.

(a) Assuming no twins, what is the probability that a family with four children will have all girls?

Answer: Since the probability of having a girl each time individually is $2/3$, we end up with a final probability of $\left(\dfrac{2}{3}\right)^4 = \dfrac{16}{81} = 0.1975$

(b) Assuming no twins, what is the probability that a family with four children will have exactly three girls?

Answer: This one is a bit more tricky. Since the probability of having a boy is $1/3$, we know that at some point we're going to go down a branch in the probability tree that gives us a $1/3$ in the final product. The rest of the branches will represent girls, and so each of those (there will be three of them) will give us a $2/3$ contributing to the final product. We also need to consider how many different ways we can do this. (I.e., BGGG, GBGG, and so on.) The easiest way to do that is to realize that we have 4 children and we want to know how many different ways 3 of them can be girls, which is given by ${}_4C_3$. In the end, we have a probability of $\left(\dfrac{1}{3}\right)\left(\dfrac{2}{3}\right)^3 {}_4C_3 = \dfrac{32}{81} = 0.3951$

(c) Assuming no twins, what is the probability that a family with four children will have exactly two girls?

Answer: $\left(\dfrac{1}{3}\right)^2\left(\dfrac{2}{3}\right)^3 {}_4C_2 = \dfrac{16}{81} = 0.1975$

(d) Assuming no twins, what is the probability that a family with four children will have two or more girls?

Answer: Adding together parts (a), (b) and (c) gives us the probability of having all girls (4 girls) or 3 girls or 2 girls... which is what we're looking for. So, the answer is $\dfrac{64}{81}$.

6. Consider the game in which you draw a card from a full deck of cards at random and are then pain a number of dollars equal to the value of the card. In this game, we consider aces to have value 1 and all face cards to have value 10.

   (a) What is the expected value of this game to the nearest cent?

   Answer: \$6.54

   (b) Suppose it costs \$7 to play the game. Is it worth it to play?

   Answer: No.

7. Suppose that 70% of the class passes this test, while 56% passed BOTH this test and the previous test. Of those who pass this test, what percentage passed the first test?

   Answer:

$$P(\text{pass 1st} \mid \text{pass 2nd}) = \frac{P(\text{pass 1st AND pass 2nd})}{P(\text{pass 2nd})} = \frac{56\%}{70\%} = 80\%$$

8. Without using your calculator, show that

   (a) ${}_{10}C_4 = 210$. (Show your work: that is, write out ${}_{10}C_4$ according to the definition and show clearly how you're simplifying to get the final answer.

   Answer:

$${}_{10}C_4 = \frac{10!}{4!(10-4)!} = \frac{10!}{4!6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2} = 10 \cdot 3 \cdot 7 = 210$$

   (b) ${}_{101}C_{99} = 101 \cdot 50$. (Hint: if I have 101 items, and I choose 99 of them to put in a box, how many items am I choosing to leave OUT of the box?)

   Answer:

$${}_{101}C_{99} = \frac{101!}{99!(101-99)!} = \frac{101!}{99!2!} = \frac{101 \cdot 100}{2} = 101 \cdot 50$$

9. Professor Slam tosses an unfair, six-sided die 200 times; the six possible numbers come up with the following frequencies.

Number on die	Frequency
1	26
2	28
3	19
4	51
5	52
6	24

Draw a histogram for the data. Label your axes as well as the numbers on the histogram itself.