Name: _____

Group: _____

Math 1120 **Rational/Irrational Numbers Worksheet** **Spring 2011**

In this worksheet, we will explore non-terminating decimals in several forms.

1. Find the decimal representation for each of the following.

   (a) $\frac{2}{7}$ $= 0.\overline{285714}$

   (b) $\frac{7}{11}$ $= 0.\overline{63}$

2. This problem is much more challenging. Can you develop a way to solve it? What is the number $2.\overline{35}$ as a fraction $\frac{a}{b}$ for integers a and b in reduced form?

   Solution: We'll write $2.\overline{35}$ as $2+S$ where $S = 0.\overline{35}$. Then,

   $$\begin{array}{rcccccccc} S & = & 0.35 & + & 0.0035 & + & 0.000035 & + & \cdots \\ \frac{1}{100}S & = & & + & 0.0035 & + & 0.000035 & + & \cdots \end{array}$$

   Subtracting $S - \frac{1}{100}S$ gives

   $$\frac{99}{100}S = 0.35 \qquad \text{and so} \qquad \frac{99}{100}S = \frac{35}{100} \qquad \text{or} \qquad S = \frac{35}{99}.$$

   Thus, the original number can be written as

   $$\begin{aligned} 2+S &= 2+\frac{35}{99} \\ &= \frac{198}{99}+\frac{35}{99} \\ &= \frac{233}{99} \end{aligned}$$

   This happens to be in reduced form.

3. Using the methods we developed in the previous problem, consider the following. A wise student asks you: "I've heard that $0.\overline{9}$ is *really* the same thing as 1... but how do I PROVE this?" Your group should develop a response to this question as best you can.

4. The same student returns a day later and says, "I have another question. I've heard that $\sqrt{2}$ cannot be written as a fraction a over b for integers a and b. Why??"