Name: _____

Math 1120 **Nonterminating Decimals Worksheet** **Spring 2011**

1. Write down an irrational number between $0.1\overline{7}$ and 0.18.

   Solution: There are MANY ways to attempt this. One way is to take your favorite irrational number, say π, and divide it by an appropriate power of ten, adding the result to $0.1\overline{7}$. If we don't divide by a large enough power of 10, then the end result will be bigger than 0.18. If we try dividing by 1000, we get

   $$0.1\overline{7} + \frac{\pi}{100} \approx 0.18091937...$$

   which is an irrational number that happens to be too large. So, we'll divide by 10,000 instead and we get

   $$0.1\overline{7} + \frac{\pi}{10000} \approx 0.178091937...$$

2. Write down an irrational number between $0.1\overline{8}$ and 0.19.

   Solution: We can approach this in the same way as the previous problem.

   $$0.1\overline{8} + \frac{\pi}{10000} \approx 0.189203047...$$

3. Write down an irrational number between $0.1\overline{9}$ and 0.20. NOTE: If you're stuck on this problem, skip it for now and go on to the next page.

   Solution: See question 6.

4. Using techniques we've discussed in class, write $0.1\overline{9}$ as a fraction in reduced form.

   Solution: Using the notation from class, we're considering $0.1\overline{9} = 0.1 + S$ where $S = 0.0\overline{9}$.

$$\begin{array}{rcccccccc} S & = 0.09 & + & 0.009 & + & 0.0009 & + & \cdots \\ \frac{1}{10}S & = & & 0.009 & + & 0.0009 & + & \cdots \end{array}$$

   Then subtracting tells us

$$S - \frac{1}{10}S = 0.09$$

$$\frac{9}{10}S = \frac{9}{100}$$

$$S = \frac{1}{10}.$$

   Hence, we know that $0.1\overline{9} = 0.1 + S = 0.1 + \frac{1}{10} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$.

5. Write 0.20 as a fraction in reduced form.

   Solution:

$$0.20 = \frac{2}{10} = \frac{1}{5}.$$

6. Now what do you think about problem 3 above?

   Solution: DOH! It's not possible, since $0.1\overline{9} = 0.2$.